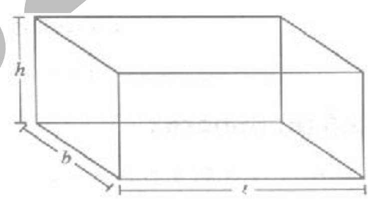
CHAPTER – 13
Surface Area and Volume

13.1 MENSURATION (SOLID FIGURES)

If any figure such as cuboids, which has three dimensions length, width and height are height are known as three dimensional figures. Where as rectangle has only two dimensional i.e., length and width. Three dimensional figures have volume in addition to areas of surface from which these soils figures are formed. Some of the main solid figures are:

(a) Cuboid:

Total Surface Area (T.S.A.) : The area of surface from which cuboid is formed. There are six faces (rectangular), eight vertices and twelve edges n a cuboid.



(i)Total Surface Area (T.S.A.) $= 2[\ell 0215b + b\times h + h\times \ell]$

(ii) Lateral Surface Area (L.A.A.) = $2[b\times h + h\times \ell]$

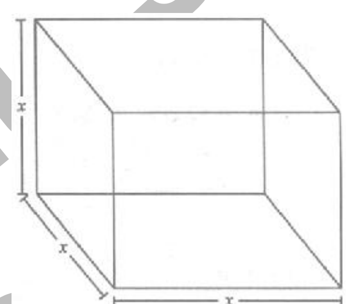
(or Area of 4 walls) $= 2h[\ell + b]$

(iii) **Volume of Cuboid** = (Area of base) × height

(iv) Length of diagonal = $\sqrt{\ell^2 + b^2 + h^2}$

(b) Cube:

Cube has six faces. Each face is a square.



(i) **T.S,A** = 2[. x + x. x + x. x]
$=2\ 2x^2 + x^2 + x^2] = 2(3x^2) = 6x^2$

(ii) **L.S.A.** $= 2\ [x^2 + x^2] = 4x^2$

(iii) **Volume** = (Area of base) × Height)
$= (x^2)\ .\ x = x^3$

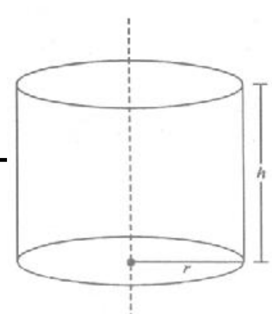
(iv) **Length of altitude** = $x\sqrt{3}$

(c) Cylinder:

Curved surface area of cylinder (C.S.A.) : It is the area of surface from which the cylinder is formed. When we cut this cylinder, we will find a rectangle with length $2\pi r$ and height h units.

(i) C.S.A. of cylinder = $(2\pi r)\times h = 2\pi rh$.

(ii) **Total Surface Area (T.S.A.) :**
T.S.A. = C.S.A. + circular top & bottom



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$= 2\pi rh + 2\pi r^2$

$= 2\pi r(h + r)$ sq. units.

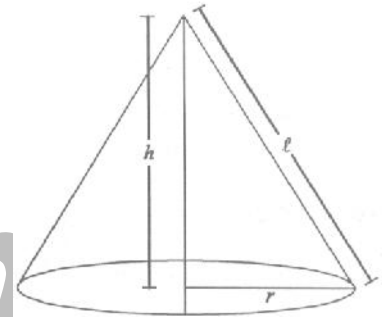
(iii) Volume of cylinder :

Volume = Area of base × height

$= (\pi r^2) \times h$

$= \pi r^2 h$ cubic units

(d) Cone:



(i) C.S.A. **=** $\pi r\ell$

(II) T.S.A. **=** C.S.A. + Other area

$= \pi r\ell$

$= \pi r(\ell + r)$

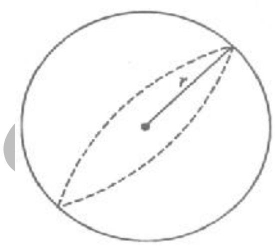
(iii) Volume $= \frac{1}{3}\pi r^2 h$

Where, h = height

r = radius of base

ℓ = slant height

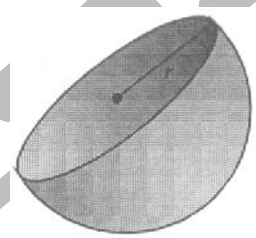
(e) Sphere:



T.S.A. = S.A. $= 4\pi r^2$

Volume $= \frac{4}{3}\pi r^3$

(f) Hemisphere:



C.S.A $= 2\pi r^2$

T.S.A = C.S.A. + other area

$= 2\pi r^2 + \pi r^2$

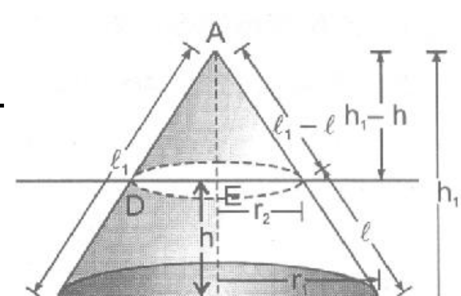
$= 3\pi r^2$

Volume = $\frac{2}{3}\pi r^3$

(g) Frustum of a Cone:

When a cone is cut by a plane parallel to base, a small cone is obtained at top and other part is obtained at bottom. This is known as **'Frustum of Cone'.**

$\Delta ABC \sim \Delta ADE$



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$\therefore \quad \frac{AC}{AE} = \frac{AB}{AD} = \frac{BC}{DE}$

$$\frac{h_1}{h_1 - h} = \frac{\ell}{\ell_1 - \ell} = \frac{r_1}{r_2}$$

Or $\frac{h_1}{h} = \frac{\ell_1}{\ell} = \frac{r_1}{r_1 - r_2}$

Volume of Frustum $= \frac{1}{3}\pi r_1^2 h_1 - \frac{1}{3}\pi {r_2}^2 (h_1 - h)$

$$= \frac{1}{3}\pi [r_1^2 h_1 - r_2^2 (h_1 - h)]$$

$$= \frac{1}{3}\pi \left[r_1^2 \left(\frac{r_1 h}{r_1 - r_2} \right) - r_2^2 \left(\frac{r_1 h}{r_1 - r_2} - h \right) \right] = \frac{1}{3}\pi h \left[\frac{r_1^3 - r_2^3}{r_1 - r_2} \right]$$

$$= \frac{1}{3}\pi h \left[r_1^2 + r_2^2 + r_1 r_2 \right]$$

Curved Surface Area of Frustum $= \pi r_1 \ell_1 - \pi r_2 (\ell_1 - \ell)$

$$= \pi \left[r_1 \left(\frac{r_1 \ell}{r_1 - r_2} \right) - r_2 \left(\frac{r_1 \ell}{r_{1-r_2}} - \ell \right) \right] = \pi \ell \left[\frac{r_1^2}{r_1 - r_2} - \frac{r_2^2}{r_1 - r_2} \right]$$

$$= \pi \ell (r_1 + r_2)$$

Total Surface Area of Frustum = CSA of frustum + $\pi r_1^2 + \pi r_1^2 + \pi r_2^2$

$$= \pi \ell (r_1 - r_2) + \pi r_1^2 + \ell r_2^2$$

Slant height of a Frustum = $\sqrt{h^2 + (r_1 - r_2)^2}$

where,

h - height of the frustum
r_1 = radius of larger circular end
r_2 = radius of smaller circular end

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