13.2 ILLUSTRATIONS

Ex.6 By melting a solid cylindrical metal, a few conical materials are to be made. If three times the radius of the cone is equal to twice the radius of the cylinder and the ratio of the height of the cylinder and the height of the cone is 4 : 3 find the number of cones which can be made.

Sol. Let R be the radius and H be the height of the cylinder and let r and h be the radius and height of the cone respectively. Then.

$3r = 2R$

and $H : h = 4 : 3$ .....(i)

$\Rightarrow \frac{H}{h} = \frac{4}{3}$

$\Rightarrow 3H = 4h$ .....(ii)

Let be the required number of cones which can be made from the material of the cylinder. The, the volume of the cylinder will be equal to the sum of the volumes of n cones. Hence, we have

$\pi R^2 H = \frac{n}{3}\pi r^2 h \Rightarrow 3R^2 H = nr^2 h$

$\Rightarrow n = \frac{3R^2H}{r^2h} = \frac{3 \times \frac{9r^2}{4} \times \frac{4h}{3}}{r^2h}$ [$\therefore$ From (i) and (ii), $R = \frac{3r}{2}$ and $H = \frac{4h}{3}$]

$\Rightarrow n = \frac{3 \times 9 \times 4}{3 \times 4}$

$\Rightarrow n = 9$

Hence, the required number of cones is 9.

Ex.7 The base diameter of solid in the form of a cone is 6 cm and the height of the cone is 10 cm. It is melted and recast into spherical balls of diameter 1 cm. Find the number of balls, thus obtained.

Sol. Let the number of spherical balls be n. Then, the volume of the cone will be equal to the sum of the volumes of the spherical balls. The radius of the base of the cone $= \frac{6}{2}$ cm = 3 cm

and the radius of the sphere $= \frac{1}{2}$cm

Now, the volume of the cone $= \frac{1}{3}\pi \times 3^2 \times 10\text{cm}^3 = 30\pi\text{cm}^3$

and, the volume of each sphere $= \frac{4}{3}\pi\left(\frac{1}{2}\right)^3 \text{cm}^3 = \frac{\pi}{6}\text{cm}^3$

Hence, we have

$n\frac{\pi}{6} = 30\pi \quad \Rightarrow \quad n = 6 \times 30 = 180$

Hence, the required number of balls = 180.

Ex.8 A conical empty vessel is to be filled up completely by pouring water into it successively with the help of a cylindrical can of diameter 6 cm and height 12 cm. The radius of the conical vessel if 9 cm and its height is 72 cm. How many times will it required to pour water into the conical vessel to fill it completely, if, in each time, the cylindrical can is filled with water completely ?

Sol. Let n be the required number of times. Then, the volume of the conical vessel will be equal to n times the volume of the cylindrical can.

Now, the volume of the conical vessel $= \frac{1}{3}\pi \times 9^2 \times 72\text{cm}^3 = 24 \times 81\pi \text{ cm}^3$

Add the volume of the cylindrical can = $\pi \times 3^2 \times 12 \text{ cm}^3 = 9 \times 12 \pi \text{ cm}^3$

Hence , $24 \times 81\pi = 9 \times 12\pi \times n$

$\Rightarrow n = \frac{24 \times 81}{9 \times 12} = 18$

Hence, the required number of times = 18.

Ex.9 The height of a right circular cylinder is equal to its diameter. It is melted and recast into a sphere of radius equal to the radius of the cylinder, find the part of the material that remained unused.

Sol. Let n be height of the cylinder. Then, its diameter is h and so its radius is $\frac{h}{2}$. Hence, its volume is

$$V_1 = \pi\left(\frac{h}{2}\right)^2 h = \frac{\pi h^3}{4}$$

Again, the radius of the sphere = $\frac{h}{2}$

Hence, the volume of the sphere is $V_2 = \frac{4}{3}\pi\left(\frac{h}{2}\right)^3 = \frac{\pi h^3}{6}$

$\therefore$ The volume of the unused material = $V_1 - V_2 = \frac{\pi h^3}{4} - \frac{\pi h^3}{6} = \frac{\pi h^3(3-2)}{12} = \frac{\pi h^3}{12} = \frac{1}{3} = \times \frac{\pi h^3}{4} = \frac{1}{3}V_1$

Hence, the required volume of the unused material is equal to $\frac{1}{3}$ of the volume of the cylinder.

Ex.10 Water flows at the rate of 10 m per minute through a cylindrical pipe having its diameter as 5 mm. How much time till it take to fill a conical vessel whose diameter of the base is 40 cm and depth 24 cm ?

Sol. Diameter of the pipe = 5 mm $\frac{5}{10}$ cm = $\frac{1}{2}$ cm.

$\therefore$ Radius of the pipe = $\frac{1}{2} \times \frac{1}{2}$ cm = $\frac{1}{4}$ cm.

In 1 minute, the length of the water column in the cylindrical pipe = 10 m = 1000 cm.

$\therefore$ Volume, of water that flows out of the pipe in 1 minute = $\pi \times \frac{1}{4} \times \frac{1}{4} \times 1000 \text{ cm}^3$.

Also, volume of the cone = $\frac{1}{3} \times \pi \times 20 \times 20 \times 24 \text{ cm}^3$.

Hence, the time needed to fill up this conical vessel = $\left(\frac{1}{3}\pi \times 20 \times 20 \times 24 \div \pi \times \frac{1}{4} \times \frac{1}{4} \times 1000\right)$ minutes

$= \left(\frac{20 \times 20 \times 24}{3} \times \frac{4 \times 4}{1000}\right) = \frac{4 \times 24 \times 16}{30}$ minutes

= $\frac{256}{5}$ minutes = 51.2 minutes.

Hence, the required time of 51.2 minutes.

Ex.11 A hemispherical tank of radius $1\frac{3}{4}$ is full of water. It is connected with a pipe which empties it at the rate of 7 liters per second. How much time will it take to empty the tank completely ?

Sol. Radius of the hemisphere = $\frac{7}{4}$ m $= \frac{7}{4} \times 100$ cm $= 175$ cm

$\therefore$ Volume of the hemisphere = $\frac{2}{3} \times \pi \times 175 \times 175 \times 175 \text{ cm}^3$

The cylindrical pipe empties it at the rate of 7 liters i.e., 7000 cm^3 of water per second.

Hence, the required time to empty the tank = $\left(\frac{2}{3} \times \frac{22}{7} \times 175 \times 175 \times 175 \div 7000\right)$s

= $\frac{2}{3} \times \frac{22}{7} \times \frac{175 \times 175 \times 175}{7000 \times 60}$ min $= \frac{11 \times 25 \times 7}{3 \times 2 \times 12}$ min $= \frac{1925}{72}$ min

$\cong$ 26.75 min, nearly.

Ex.12 A well of diameter 2 m is dug 14 m deep. The earth taken out of its is spread evenly all around it to a width of 5 m to from an embankment. Find the height of the embankment.

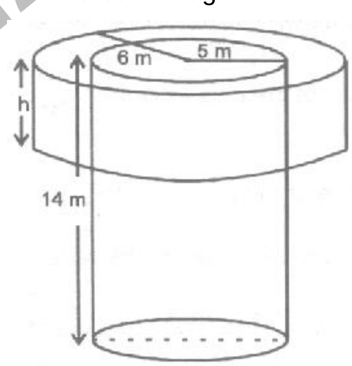
Sol. Let n be the required height of the embankment.
The shape of the embankment will be like the shape of a cylinder of internal radius 1 m and external radius (5 + 1) m = 6 m [figure].
The volume of the embankment will be equal to the volume of the earth dug out from the well. Now, the volume of the earth = volume of the cylindrical well

= $\pi \times 1^2 \times 14 \text{ m}^3$

= 14 $\pi \text{ m}^3$

Also, the volume of the embankment
= $\pi(6^2 - 1^2)$ h cm^3 = 35 π h m^3
Hence, we have
35 πh = 14 π

$\Rightarrow$ h $= \frac{14}{35} = \frac{2}{5} = 0.4$



Hence, the required height of the embankment = 0.4 m

Ex.13 Water in a canal, 30 dm wide and 12 dm deep, is flowing with a speed of 10 km/hr. How much area will it irrigate in 30 minutes if 8 cm of standing water is required from irrigation.

Sol. Speed of water in the canal = 10 km. h = 10000 m.60 min = $\frac{500}{3}$ m/min.

$\therefore$ The volume of the water flowing out of the canal in 1 minute = $\left(\frac{500}{3}\times\frac{30}{10}\times\frac{12}{10}\right)$ m^2 = 600 m^3

$\therefore$ In 30 min, the amount of water flowing out of the canal = (600 × 30) m^3 = 600 m^3

If the required area of the irrigated land is × m^2, then the volume of water to be needed to irrigate the land

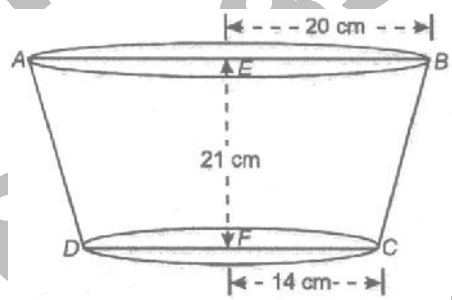
$= \left(x \times \frac{8}{100}\right) m^3 \qquad = \frac{2x}{25} m^3$

Hence, $\frac{2x}{25} = 18000 \qquad \Rightarrow \qquad x = 18000 \times \frac{25}{2} = 225000$

Hence, the required area is 225000 m^2.

Ex.14 A bucket is 40 cm in diameter at the top and 28 cm in diameter at the bottom. Find the capacity of the bucket in litters, if it is 21 cm deep. Also, find the cost of tin sheet used in making the bucket, if the cost of tin is Rs. 1.50 per sq dm.

Sol. Given : r_1 = 20 cm r_2 = 14 cm and h = 21 cm



Now, the required capacity (i.e. volume) of bucket = $\frac{\pi h}{3}(r_1^2 + r_1 r_2 + r_2^2)$

$\cong \frac{22\times 21}{7\times 3}(20^2 + 20\times 14 + 14^2)$ cm^3 = 22 × 876 cm^3 = 19272 cm^3 = $\frac{19272}{1000}$ liters = 19.272 liters.

Now, l = $\sqrt{(r_1 - r_2)^2 + h^2} = \sqrt{(20-14)^2 + 21^2}$ cm $= \sqrt{6^6 + 21^2}$ cm $= \sqrt{36+441}$ cm $= \sqrt{477}$ cm $\cong 21.84$ **cm.**

$\therefore$ Total surface area of the bucket (which is open at the top)

$= \pi\ell(r_1 + r_2) + \pi r_2^2$

$= \pi[(r_1 + r_2)\ell + r_2^2]$

$= \frac{22}{7}\left[(20+14)21.84 + 14^2\right]$

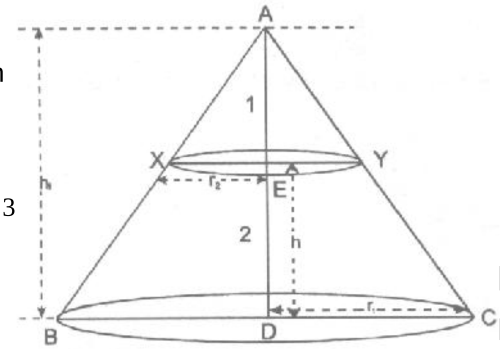
= 2949.76 cm^3

$\therefore$ Required cost of the tin sheet at the rate of Rs. 1.50 per dm^2 i.e., per 100 cm^2

$= \text{Rs } \dfrac{1.50 \times 2949.76}{100} \cong \text{Rs } 44.25$

Ex.15 A cone is divided into two parts by drawing a plane through a point which divides its height in the ratio 1 : 2 starting from the vertex and the place is parallel to the base. Compare the volume of the two parts.

Sol. Let the plane XY divide the cone ABC in the ratio AE : ED = 1 : 2, where AED is the axis of the cone. Let r_2 and r_2 be the radii of the circular section XY and the base BC of the cone respectively and let h_1 - h and h_1 be their heights **[figure].**



Then, $\dfrac{h_1}{h} = \dfrac{3}{2} \Rightarrow h = \dfrac{3}{2}h$

And $\dfrac{r_1}{r_2} = \dfrac{h_1}{h_1 - h} = \dfrac{\frac{3}{2}h}{\frac{1}{2}h} = 3$

$\therefore \quad r_1 = 3r_2$

Volume of cone AXY

$$= \frac{1}{32}\pi r_2^2 (h_1 - h)$$

$$= \frac{1}{3}\pi r_2^2 \left(\frac{3}{2}h - h\right)$$

$$= \frac{1}{6}\pi r_2^2 h$$

Volume of frustum XYBC

$$= \frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$$

$$= \frac{1}{3}\pi h(9r_2^2 + r_2^2 + 3r_2^2)$$

$$= \frac{1}{3}\pi h(13r_2^2)$$

So, $$\frac{\text{Volume of cone AXY}}{\text{Volume of frustum XYBC}} = \frac{\frac{1}{6}\pi r_2^2 h}{\frac{13}{3}\pi r_2^2 h}$$

$$\frac{\text{Volume of cone AXY}}{\text{Volume of frustum XYBC}} = \frac{1}{26}.$$

i.e. the ratio between the volume of the cone AXY and the remaining portion BCYX is 1 : 26.