2.3 RELATIONSHIP BETWEEN ZEROS AND COEFFICIENTS OF QUADRATIC POLYNOMIA

Let α and β be the zeros of a quadratic polynomial $f(x) = ax^2 + bx + c$. By facto r theorem $(x-\alpha)$ and $(x-\beta)$ are the factors of f(x).

$\therefore \quad f(x) = k\ (x-\alpha)(x-\beta)$ are the factors of f(x)

$\Rightarrow \quad ax^2 + bx + c = k\{x^2 - (\alpha+\beta)x + \alpha\beta\}$

$\Rightarrow \quad ax^2 + bx + c = kx^2 - k(\alpha+\beta)x + k\alpha\beta$

Comparing the coefficients of x^2, x and constant terms on both sides, we get

$a = k,\ b = -k\ (\alpha+\beta)$ and $k\alpha\beta$

$\Rightarrow \quad \alpha+\beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

$\Rightarrow \quad \alpha+\beta = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$ and $\alpha\beta = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

Hence,

Sum of the zeros $= -\frac{b}{a} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

Product of the zeros $= \frac{c}{a} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

REMAKRS :

If α and β are the zeros of a quadratic polynomial f(x). The , the polynomial f(x) is given by

$f(x) = k\{x^2 - (\alpha+\beta)x + \alpha\beta\}$

or $\quad$ f(x) = k{x^2 - (Sum of the zeros) x + Product of the zeros}

Ex.5 Find the zeros of the quadratic polynomial $f(x) = x^2 - 2x - 8$ and verify and the relationship between the zeros and their coefficients.

Sol. $f(x) = x^2 - 2x - 8$

$\Rightarrow \quad f(x) = x^2 - 4x + 2x - 8 \qquad \Rightarrow \qquad f(x) = x(x - 4) + 2(x - 4)]$

$\Rightarrow \quad f(x) = (x - 4)\ (x + 2)$

Zeros of f(x) are given by f(x) = 0

$\Rightarrow \quad x^2 - 2x - 8 = 0 \qquad \Rightarrow \qquad (x - 4)\ (x + 2) = 0$

$\Rightarrow \quad x = 4$ or $x = -2$

So, $\alpha = 4$ and $\beta = -2$

$\therefore$ sum of zeros $\alpha+\beta$

$= 4 - 2 = 2$

Also, sum of zeros $= -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = \frac{-(-2)}{1} = 2$

So, sum of zeros $= \alpha+\beta = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

Now, product of zeros $= \alpha\beta$

$= (4)\ (-2) = -8$

Also, product of zeros $= \dfrac{\text{Constant term}}{\text{Coefficient of } x^2} = \dfrac{-8}{1} = -8$

$\therefore$ Product of zeros $= \dfrac{\text{Constant term}}{\text{Coefficient of } x^2} = \alpha\beta$.

Ex.6 Find a quadratic polynomial whose zeros are $5 + \sqrt{2}$ and $5 - \sqrt{2}$

Sol. Given $\alpha = 5 + \sqrt{2}, \beta = 5 - \sqrt{2}$

$\therefore$ $f(x) = k\{x^2 - x(\alpha + \beta) + \alpha\beta\}$

Here, $\alpha + \beta = 5 + \sqrt{2} + 5 - \sqrt{2} = 10$

and $\alpha\beta = (5 + \sqrt{2})(5 - \sqrt{2})$

$= 25 - 2 = 23$

$\therefore$ $f(x) = k\{x^2 - 10x + 23\}$, where, k is any non-zero real number.

Ex.7 Sum of product of zeros of quadratic polynomial are 5 and 17 respectively. Find the polynomial.

Sol. Given : Sum of zeros = 5 and product of zeros = 17

So, quadratic polynomial is given by

$\Rightarrow$ $f(x) = k\{x^2 -$ x(sum of zeros) + product of zeros$\}$

$\Rightarrow$ $f(x) = k\{x^2 - 5x + 17\}$, where, k is any non-zero real number,