2.3 RELATIONSHIP BETWEEN ZEROS AND COEFFICIENTS OF QUADRATIC POLYNOMIA

Let α and β be the zeros of a quadratic polynomial $f(x) = ax^2 + bx + c$. By facto r theorem $(x - \alpha)$ and $(x - \beta)$ are the factors of f(x).

$$f(x) = k(x - \alpha)(x - \beta)$$
 are the factors of $f(x)$

$$\Rightarrow$$
 ax² + bx + c = k{x² - (\alpha + \beta)x + \alpha \beta}

$$\Rightarrow$$
 $ax^2 + bx + c = kx^2 - k(\alpha + \beta)x + k\alpha\beta$

Comparing the coefficients of x^2 , x and constant terms on both sides, we get $a = k, b = -k (\alpha + \beta)$ and $k\alpha\beta$

$$\Rightarrow \alpha + \beta = -\frac{b}{a} \text{ and } \alpha \beta = \frac{c}{a}$$

$$\Rightarrow \alpha + \beta = -\frac{b}{a} \text{ and } \alpha \beta = \frac{c}{a}$$

$$\Rightarrow \alpha + \beta = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} \text{ and } \alpha \beta = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

Hence,

Sum of the zeros =
$$-\frac{b}{a}$$
 = $-\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

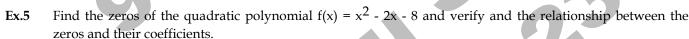
Product of the zeros =
$$\frac{c}{a} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

REMAKRS:

If α and β are the zeros of a quadratic polynomial f(x). The , the polynomial f(x) is given by

$$\mathbf{f}(\mathbf{x}) = \mathbf{k} \{ \mathbf{x}^2 - (\alpha + \beta)\mathbf{x} + \alpha\beta \}$$

or
$$f(x) = k\{x^2 - (Sum of the zeros) x + Product of the zeros\}$$



Sol.
$$f(x) = x^2 - 2x - 8$$

$$\Rightarrow$$
 f(x) = x² - 4x + 2x - 8 \Rightarrow f(x) = x(x - 4)

$$\Rightarrow$$
 f(x) = (x - 4) (x + 2)

Zeros of f(x) are given by f(x) = 0

$$\Rightarrow x^2 - 2x - 8 = 0$$

$$\Rightarrow (x-4)(x+2) =$$

$$\Rightarrow$$
 x = 4 or x = -2

So,
$$\alpha = 4$$
 and $\beta = -2$

$$\therefore$$
 sum of zeros $\alpha + \beta$

Also, sum of zeros =
$$-\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = \frac{-(-2)}{1} = 2$$

So, sum of zeros =
$$\alpha + \beta = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

Now, product of zeros =
$$\alpha\beta$$



Also, product of zeros =
$$\frac{\text{Cons tan t term}}{\text{Coefficient of } x^2} = \frac{-8}{1} = -8$$

$$\therefore \qquad \text{Product of zeros} = \frac{\text{Cons tan t term}}{\text{Coefficient of } x^2} = \alpha \beta.$$

Ex.6 Find a quadratic polynomial whose zeros are $5 + \sqrt{2}$ and $5 - \sqrt{2}$

Sol. Given
$$\alpha = 5 + \sqrt{2}$$
, $\beta = 5 - \sqrt{2}$

$$\therefore f(x) = k\{x^2 - x(\alpha + \beta) + \alpha\beta\}$$

Here,
$$\alpha + \beta = 5 + \sqrt{2} + 5 - \sqrt{2} = 10$$

and
$$\alpha\beta = (5 + \sqrt{2})(5 - \sqrt{2})$$

- $f(x) = k \{x^2 10x + 23\}$, where, k is any non-zero real number.
- **Ex.7** Sum of product of zeros of quadratic polynomial are 5 and 17 respectively. Find the polynomial.
- **Sol.** Given: Sum of zeros = 5 and product of zeros = 17
 - So, quadratic polynomial is given by
 - \Rightarrow f(x) = k {x² x(sum of zeros) + product of zeros}
 - \Rightarrow f(x) = k{x² 5x + 17}, where, k is any non-zero real number,

