2.5 FINDING REMAINNING ZEROES OF A POLYNOMIAL IF SOME ARE GIVEN

VALUE OF A POLYNOMIAL:

The value of a polynomial **f(x)** at **x =** α is obtained by substituting $x = \alpha$ in the given polynomial and is denoted by $f(\alpha)$.

For example: If $f(x) = 2x^3 - 13x^2 + 17x + 12$ then its value at x = 1 is

$f(1) = 2(1)^3 - 13(1)^2 + 17(1) + 12 = 2 - 13 + 17 + 12 = 18.$

ZEROS OR ROOTS OF A POLYNOMIAL:

A real number **'a'** is a zero of a polynomial **f(x),** if **f(a) = 0,** Here **'a'** is called a root of the equation **f(x) = 0.**

Ex.10 Show that x = 2 is a root of $2x^3 + x^2 - 7x - 6$

Sol. $p(x) = 2x^3 + x^2 - 7x - 6.$

Then, $p(2) = 2(2)^3 + (2)^2 - 7(2) - 6 = 16 + 4 - 14 = 0$

Hence x = 2 is a root of p(x).

Ex.11 If $x = \frac{4}{3}$ is a root of the polynomial $f(x) = 6x^3 - 11x^2 + kx - 20$ then find the value of k.

Sol. $f(x) = 6x^3 - 11x^2 + kx - 20$

$f\left(\frac{4}{3}\right) = 6\left(\frac{4}{3}\right)^3 - 11\left(\frac{4}{3}\right)^2 + k\left(\frac{4}{3}\right) - 20 = 0$

$\Rightarrow \quad 6\left(\frac{64}{27}\right) - 11\left(\frac{16}{9}\right) + \frac{4k}{3} - 20 = 0 \Rightarrow 6\left(\frac{64}{27}\right) - 11\left(\frac{16}{9}\right) + \frac{4k}{3} - 20 = 0$

$\Rightarrow$ 128 - 176 + 12k - 180 = 0

$\Rightarrow$ 12k + 128 - 356 = 0

$\Rightarrow$ 12k = 228

$\Rightarrow$ k = 19.

Ex.12 If x = 2 & x = 0 are roots of the polynomials $(f)x = 2x^3 - 5x^2 + ax + b$, then find the values of a and b/

Sol. $f(2) = 2(2)^3 - 5(2)^2 + a(2) + b = 0$

$\Rightarrow$ 16 - 20 + 2a + b = 0 $\Rightarrow$ 2a + b = 4 .....(i)

$\Rightarrow$ $f(0) = 2(0)^3 - 5(0)^2 + a(0) + b = 0 \Rightarrow$ b = 0

$\Rightarrow$ 2a = 4 $\Rightarrow$ a = 2, b = 0.

FACTOR THEOREM:

Let **p(x)** be a polynomial of degree greater than or equal to 1 and **'a'** be a real number such that **p(a) = 0.** Then **(x - a)** is a factor of **p(x).** Conversely, if **(x - a)** is a factor of **p(x)**, then **p(a) = 0.**

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Ex.13 Show that x + 1 and 2x - 3 are factors of $2x^3 - 9x^2 + x + 12$.

Sol. To prove that (x + 1) and (2x - 3) are factors of $p(x) = 2x^3 - 9x^2 + x + 12$ it is sufficient to show that p(-1) and $p\left(\frac{3}{2}\right)$ both are equal to zero.

$p(-1) = 2(-1)^3 - 9(-1)^2 + (-1) + 12 = - 2 - 9 - 1 + 12 = - 12 + 12 = 0$

And $p\left(\frac{3}{2}\right) = 2\left(\frac{3}{2}\right)^3 - 9\left(\frac{3}{2}\right)^2 + \left(\frac{3}{2}\right) + 12$

$= \frac{27}{4} - \frac{81}{4} + \frac{3}{2} + 12 = \frac{27 - 81 + 6 + 48}{4} = \frac{-81 + 81}{4} = 0$

Ex.14 Find α and β if x + 1 and x + 2 are factors of $p(x) = x^3 + 3x^2 - 2\alpha x + \beta$.

Sol. x + 1 and x + 2 are the factor of p(x).

Then, p(-1) = 0 & p(-2) = 0

Therefore, $p(-1) = (-1)^3 + 3(-1)^2 - 2\alpha(-1) + \beta = 0$

$\Rightarrow \quad -1 + 3 + 2\alpha + \beta = 0 \Rightarrow \beta = -2\alpha - 2$ ....(i)

$p(-2) = (-2)^3 + 3(-2)^2 - 2\alpha(-2) + \beta = 0$

$\Rightarrow \quad -8 + 12 + 4\alpha + \beta = 0 \Rightarrow \beta = -4\alpha - 4$ ....(ii)

From equation (1) and (2)

$-2\alpha - 2 = -4\alpha - 4$

$\Rightarrow \quad 2\alpha = -2 \Rightarrow \alpha = -1$

Put α = -1 equation (1) $\Rightarrow$ β = - 2 (-1) - 2 = 2 - 2 = 0.

Hence $\alpha = -1, \beta = 0$

Ex.15 What must be added to $3x^3 + x^2 - 22x + 9$ so that the result is exactly divisible by $3x^2 + 7x - 6$.

Sol. Let $p(x) = 3x^3 + x^2 - 22x + 9$ and $q(x) = 3x^2 + 7x - 6$

We know if p(x) is divided by q(x) which is quadratic polynomial then the remainder be r(x) and degree of r(x) is less than q(x) or Divisor.

$\therefore$ By long division method

Let we added ax + b (linear polynomial) in p(x), so that p(x) + ax + b is exactly divisible by $3x^2 + 7x - 6$.

Hence, $p(x) + ax + b = s(x) = 3x^3 - x^2 - 22x + 9 + ax + b = 3x^3 + x^2 x(22 - a) + (9 + b)$.

$$\begin{array}{r|l} & x-2 \\ 3x^2+7x-6 & 3x^3 + x^2 - x(22-a) + 9 + b \\ & -3x^3 \pm 7x^2_{+} - 6x \\ \hline & -6x^2 + 6x - (22-a)x + 9 + b \\ & -6x^2 x(-16+a) + 9 + b \\ & {}_{+}-6x^2_{+} - 14x \pm 12 \\ \hline & x(-2+a) + (b-3) = 0 \end{array}$$

Hence, x(a - 2) + b - 3 = 0. x + 0

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⇒ a - 2 = 0 & b - 3 = 0

⇒ a = 2 and b = 3

Hence if in p(x) we added 2x + 3 then it is exactly divisible by $3x^2 + 7x - 6$.

Ex.16 What must be subtracted from $x^3 - 6x^2 - 15x + 80$ so that the result is exactly divisible by $^2 + x - 12$.

Sol. Let ax + b be subtracted from $p(x) = x^3 - 6x^2 - 15x + 80$ so that it is exactly divisible by $x^2 + x - 12$.

∴ $s(x) = x^3 - 6x^2 - 15x + 80 - (ax + b)$

$= x^3 - 6x^2 - (15 + a)x + (80 - b)$

Dividend = Divisor × quotient + remainder

But remainder will be zero.

∴ Dividend = Divisor × quotient

⇒ $s(x) = (x^2 + x - 12)$ × quotient

⇒ $s(x) = x^3 - 6x^2 - (15 + a)x + (80 - b)$

$$\begin{array}{r|l} & x-7 \\ x^2+x-12 & x^3-6x^2-x(15+a)+80-b \\ & \underline{{}_{-}x^3 \pm x^2 \mp 12x} \\ & -7x^2+12x-(15+a)x+80-b \\ & -7x^2+(-3-a)+80-b \\ & \underline{{}_{\mp}7x^2 {}_{\mp}7x \qquad \pm 84} \\ & x(4-a)+(-4-b)=0 \end{array}$$

Hence, x(4 - a) + (-4 -b) = 0.x + 0

⇒ 4 - a = 0 & (-4 - b) = 0

⇒ a = 4 and b = - 4

Hence, if in p(x) we subtract 4x - 4 then it is exactly divisible by $x^2 + x - 12$.

Ex.17 Using factor theorem, factorize : $p(x) = 2x^4 - 7x^3 - 13x^2 + 63x - 45$.

Sol. 45 ⇒ ±1,±3,±5,±9,±15,±45

If we put x = 1 in p(x)

$p(1) = 2(1)^4 - 7(1)^3 - 13(1)^2 + 63(1) - 45$

p(1) = 2 - 7 - 13 + 63 - 45 = 65 - 65 = 0

∴ x = 1 or x - 1 is a factor of p(x).

Similarly if we put x = 3 in p(x)

$p(3) = 2(3)^4 - 7(3)^3 - 13(3)^2 + 63(3) - 45$

p(3) = 162 - 189 - 117 + 189 - 45 = 162 - 162 = 0

Hence, x = 3 or (x - 3) = 0 is the factor of p(x).

$p(x) = 2x^4 - 7x^3 - 13x^2 + 63x - 45$

∴ $p(x) = 2x^3 (x - 1) - 5x^2 (x - 1) - 18x(x - 1) + 45(x - 1)$

⇒ $p(x) = (x - 1) (2x^3 - 5x^2 - 18x + 45)$

⇒ $p(x) = (x - 1) (2x^3 - 5x^2 - 18x + 45)$

⇒ $p(x) = (x - 1) [2x^2 -(x - 3) + x(x - 3) - 15(x - 3)]$

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$\Rightarrow$ p(x) = (x - 1) (x - 3) $(2x^2 + x - 15)$

$\Rightarrow$ p(x) = (x - 1) (x - 3) $(2x^2 + 6x - 5x - 15)$

$\Rightarrow$ p(x) = (x - 1) (x - 3)[2x(x + 3) - 5(x + 3)]

$\Rightarrow$ p(x) = (x - 1) (x - 3) (x + 3) (2x - 5).

REMAINDER THEOREM :

Let **p(x)** be any polynomial of degree greater than or equal to one and **'a'** be any real number. If **p(x)** is divided by **x - a),** then the remainder is equal to **p(a).**

Let q(x) be the quotient and r(X) be the remainder when p(x) is divided by (x - a), then

Dividend = Divisor × Quotient + Remainder

Ex.18 Find the remainder when $f(x) = x^3 - 6x^2 + 2x - 4$ is divided by g(x) = 1 - 2x.

Sol. $1 - 2x = 0 \Rightarrow 2x = 1 \Rightarrow x = \frac{1}{2}$

$$f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 - 6\left(\frac{1}{2}\right)^2 + 2\left(\frac{1}{2}\right) - 4 = \frac{1}{8} - \frac{3}{2} + 1 - 4 = \frac{1 - 12 + 8 - 32}{8} = -\frac{35}{8}$$

Ex.19 Apply division algorithm to find the quotient q(x) and remainder r(x) on dividing $f(x) = 10x^4 + 17x^3 - 62x^2 + 30x - 3$ by $b(x) = 2x^2 - x + 1$.

Sol.

$$\begin{array}{r|l} & 5x^2 + 11x - 28 \\ \hline 2x^2 - x + 1 & 10x^4 + 17x^3 - 62x^2 + 30x - 3 \\ & \underset{-}{}10x^4 \underset{+}{-} 5x^3 \underset{-}{+} 5x^2 \\ \hline & 22x^3 - 67x^2 + 30x - 3 \\ & \underset{-}{}22x^3 \underset{+}{-} 11x^2 \underset{-}{+} 11x \\ \hline & -56x^2 + 19x - 3 \\ & \underset{+}{-}56x^2 \underset{-}{+} 28x \underset{+}{-} 28 \\ \hline & -9x + 25 \end{array}$$

So, quotient $q(x) = 5x^2 + 11x - 28$ and remainder r(x) = -9x + 25 .

Now, dividend = Quotient × Divisor + Remainder

$= (5x^2 + 11x - 28)(2x^2 - x + 1) + (-9x + 25)$

$= 10x^4 - 5x^3 + 5x^2 + 22x^3 - 11x^2 + 11x - 56x^2 + 28x - 28 - 9x + 25$

$= 10x^4 + 17x^3 - 62x^2 + 30x - 3$

Hence, the division algorithm is verified.

Ex.20 Find all the zeros of the polynomial $f(x) = 2x^4 - 2x^3 - 7x^2 + 3x + 6$, if two of its zeros are $-\sqrt{\frac{3}{2}}$ and $\sqrt{\frac{3}{2}}$.

Sol. Since $-\sqrt{\frac{3}{2}}$ and $\sqrt{\frac{3}{2}}$ are zeros of f(x).

Therefore, $\left(x + \sqrt{\frac{3}{2}}\right)\left(x - \sqrt{\frac{3}{2}}\right) = \left(x^2 - \frac{3}{2}\right) = \frac{2x^2 - 3}{2}$ or $2x^2 - 3$ is a factor of f(x).

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$$
\begin{array}{r|l}
 & x^2 - x - 2 \\
\hline
2x^2 - 3 & 2x^4 - 2x^3 - 7x^2 + 3x + 6 \\
 & \underline{\underset{-}{}2x^4 \qquad \underset{\mp}{}3x^2} \\
 & -2x^3 - 4x^2 + 3x + 6 \\
 & \underline{\underset{\mp}{}2x^3 \qquad \underset{\mp}{}3x} \\
 & -4x^2 + 6 \\
 & -4x^2 + 6 \\
 & \underline{+ \qquad -} \\
 & 0
\end{array}
$$

$\therefore\ 2x^4 - 2x^3 - 7x^2 + 3x + 6 = (2x^2 - 3)(x^2 - x - 2)$

$= (2x^2 - 3)(x - 2)(x + 1)$

$= 2\left(x + \sqrt{\frac{3}{2}}\right)\left(x - \sqrt{\frac{3}{2}}\right)(x - 2)(x + 1)$

So, the zeros are $-\sqrt{\frac{3}{2}}, \sqrt{\frac{3}{2}}, 2, -1$

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