3.2 METHODS TO SOLVE A PAIR OF EQUATION

(1) Elimination by Substitution:

Ex.1 Solve $x + 4y = 14$ .....(i)

$7x - 3y = 5$ ....(ii)

Sol. From equation (i) $x = 14 - 4y$ ....(iii)

Substitute the value of x in equation (ii)

$\Rightarrow \quad 7(14 - 4y) - 3y = 5$

$\Rightarrow \quad 98 - 28y - 3y = 5$

$\Rightarrow \quad 98 - 31y = 5$

$\Rightarrow \quad 93 = 31y$

$\Rightarrow \quad y = \frac{93}{31} \Rightarrow y = 3$

Now substitute value of y in equation (iii)

$\Rightarrow \quad 7x - 3(3) = 5$

$\Rightarrow \quad 7x - 3(3) = 5$

$\Rightarrow \quad 7x = 14$

$\Rightarrow \quad x = \frac{14}{7} = 2$

So, solution is x = 2 and y = 3

(2) Elimination by Equating the Coefficients:

Ex.2 Solve $9x - 4y = 8$ .....(i)

$13x + 7y = 101$ ....(ii)

Sol. Multiply equation (i) by 7 and equation (ii) by 4, we get

Add $\quad 63x - 28y = 56$

$\quad 52x + 28y = 404$

$\quad 115x = 460$

$\Rightarrow \quad x = \frac{460}{115} \Rightarrow x = 4$.

Substitute x = 4 in equation (i)

$9(4) - 4y = 8 \quad \Rightarrow \quad 36 - 8 = 4y \quad \Rightarrow \quad 28 = 4y \Rightarrow \quad y = \frac{28}{4} = 7$

So, solution is x = 4 and y = 7.

(3) Elimination by Cross Multiplication:

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0 \qquad \left[\because \frac{a_1}{a_2} \neq \frac{b_1}{b_2}\right]$

$$\begin{matrix} b_1 & & c_1 & & a_1 & & b_1 \\ & \times & & & & \times & \\ b_2 & & c_2 & & a_2 & & b_2 \end{matrix}$$ [Write the coefficient in this manner]

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{a_2c_1 - a_1c_2} = \frac{1}{a_1b_2 - a_2b_1} \Rightarrow \therefore \frac{x}{b_1c_2 - b_2c_1} = \frac{1}{a_1b_2 - a_2b_1}$$

$$\Rightarrow \quad x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$$

Also, $\quad \dfrac{y}{a_2c_1 - a_1c_2} = \dfrac{1}{a_1b_2 - a_2b_1}$

$$\therefore \quad y = \frac{a_2c_1 - a_1c_2}{a_1b_2 - a_2b_1}$$

Ex.3 Solve $3x + 2y + 25 = 0$ .....(i)

$x + y + 15 = 0$ ....(ii)

Sol. Here, $a_1 = 3$ $b_1 = 2$, $c_1 = 25$

$a_2 = 1$ $b_2 = 1$, $c_2 = 15$

$$\therefore \begin{matrix} 2 & & 25 & & 3 & & 2 \\ & \times & & \times & & \times & \\ 1 & & 15 & & 1 & & 1 \end{matrix}$$

$$\frac{x}{2\times15-25\times1} = \frac{y}{25\times1-15\times3} = \frac{1}{3\times1-2\times1}; \frac{x}{30-25} = \frac{y}{25-45} = \frac{1}{3-2}$$

$$\frac{x}{5} = \frac{y}{-20} = \frac{1}{1} \quad .....(i)$$

$$\frac{x}{5} = 1, \frac{y}{-20} = \frac{1}{1}$$

X = 5, y = - 20

So, solution is x = 5 and y = - 20.