3.3 CONDITIONS FOR SOLVABILITY (OR CONSISTENCY) OF SYSTEM OF EQUATIONS

(a) Unique Solution :

Two lines $a_1 + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, if the denominator $a_1b_2 - a_2b_1 \neq 0$ then the given system of equations have unique solution (i.e. only one solution) and solutions are said to be consistent.

$\therefore \quad a_1b_2 - a_2b_1 \neq 0 \quad \Rightarrow \quad \frac{a_1}{b_2} \neq \frac{b_1}{b_2}$

(b) No Solution :

Two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, if the denominator $a_1b_2 - a_2b_1 = 0$ then the given system of equations have no solution and solutions are said to be consistent.

$\therefore \quad a_1b_2 - a_2b_1 \neq 0 \Rightarrow \quad \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

(c) Many Solution (Infinite Solutions)

Two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = -$ then system of equations has many solution and solutions are said to be consistent.

Ex.4 Find the value of 'P' for which the given system of equations has only one solution (i.e. unique solution).

$Px - y = 2$ ....(i)

$6x - 2y = 3$ ....(ii)

Sol. $a_1 = P, b_1 = -1, c_1 = -2$

$a_2 = 6\ b_2 = -2, c_2 = -3$

Conditions for unique solution is $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\Rightarrow \quad \frac{P}{6} \neq \frac{+1}{+2} \quad \Rightarrow \quad P \neq \frac{6}{2} \quad \Rightarrow \quad P \neq 3$

$\therefore$ P can have all real values except 3.

Ex.5 Find the value of k for which the system of linear equation

$kx + 4y = k - 4$

$16x + ky = k$ has infinite solution.

Sol. $a_1 = k, b_1 = 4, c_1 = -(k - 4)$

$a_2 = 16, b_2 = k, c_2 = -k$

Here condition is $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\Rightarrow \quad \frac{k}{16} = \frac{4}{k} = \frac{(k-4)}{(k)}$

$\Rightarrow \quad \frac{k}{16} = \frac{4}{k}$ also $\frac{4}{k} = \frac{k-4}{k}$

$\Rightarrow \quad k^2 = 64 \qquad \Rightarrow \qquad 4k = k^2 - 4k$

$\Rightarrow \quad k = \pm\ 8 \qquad \Rightarrow \qquad k(k-8) = 0$

k = 0 or k = 8 but k = 0 is not possible other wise equation will be one variable.

$\therefore$ k = 8 is correct value for infinite solution.

Ex.6 Determine the value of k so that the following linear equations has no solution.

$(3x + 1)\ x + 3y - 2 = 0$

$(k^2 + 1)\ x + (k - 2)\ y - 5 = 0$

Sol. Here $a_1 = 3k + 1$, $b_1 = 3$ and $c_1 = -2$

$a_2 = k^2 + 1$, $b_2 = k - 2$ and $c_2 = -5$

For no solution, condition is $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\frac{3k+1}{k^2+1} = \frac{3}{k-2} \neq \frac{-2}{-5}$

$\Rightarrow \quad \frac{3k+1}{k^2+1} = \frac{3}{k-2}$ and $\frac{3}{k-2} \neq \frac{2}{5}$

Now, $\frac{3k+1}{k^2+1} = \frac{3}{k-2}$

$\Rightarrow \quad (3k + 1)\ (k - 2) = 3(k^2 + 1)$

$\Rightarrow \quad 3k^2 - 5k - 2 = 3k^2 + 3$

$\Rightarrow \quad -5k - 2 = 3$

$\Rightarrow \quad -5k = 5$

$\Rightarrow \quad k = -1$

Clearly, $\frac{3}{k-2} \neq \frac{2}{5}$ for k = - 1.

Hence, the given system of equations will have no solution for k = - 1.