4.2 ROOTS OF A QUADRATIC EQUATION

The value of x which satisfies the given quadratic equation is known as its root. The roots of the given equation are known as its solution.

General form of a quadratic equation is :

$$ax^2 + bx + c = 0$$

or $4a^2x^2 + 4abx + 4ac = -4ac$ [Multiplying by 4a]

or $4a^2x^2 + 4abx = -4ac$ [By adding b^2 both sides]

or $4a^2x^2 + 4abc + b^2 = b^2 - 4ac$

or $(2ax + b)^2 = b^2 - 4ac$

Taking square root of both the sides

$$2ax + b = \pm\sqrt{b^2 - 4ac}$$

or $$x = \frac{-b \pm \sqrt{b2} - 4ac}{2a}$$

Hence, roots of the quadratic equation $ax^2 + bx + c = 0$ are $\frac{-b+\sqrt{b^2-4ac}}{2a}$ and $\frac{-b-\sqrt{b^2-4ac}}{2a}$

REMARK:

A quadratic equation is satisfied by exactly two values of 'a' which may be real or imaginary. The equation, ax2 + bx + c = 0 is :

A quadratic equation if $a \neq 0$ Two roots

A linear equation if a = 0, $b \neq 0$ One root

A contradiction if a = b = 0, $c \neq 0$ No root

An identify if a = b = c = 0 Infinite roots

A quadratic equation cannot have more than two roots.

It follows from the above statement that if a quadratic equation is satisfied by more than two values of x, then it is satisfied by every value of x and so it is an identity.