# 4.3 NATURE OF ROOTS

Consider the quadratic equation, $ax^2 + bx + c = 0$ having $\alpha\beta$ as its roots and $b^2 - 4ac$ is called discriminate of roots of quadratic equation. It is denoted by D or $\Delta$.

Roots of the given quadratic equation may be

(i) Real and unequal (ii) Real and equal (iii) Imaginary and unequal.

Let the roots of the quadratic equation $ax^2 + bx + c = 0$ (where $a \neq 0, b, c \in R$ ) be $\alpha$ and $\beta$ then

$$\alpha = \frac{-b+\sqrt{b^2-4ac}}{2a} \quad .....(i)$$

and $$\beta = \frac{-b-\sqrt{b^2-4ac}}{2a} \quad ....(ii)$$

The nature of roots depends upon the value of expression **'$b^2$ - 4ac'** with in the square root sign. This is known as discriminate of the given quadratic equation.

**Consider the Following Cases :**

**Case-1 When $b^2 - 4ac > 0$, (D > 0)**

In this case roots of the given equation are real and distinct and are as follows

$$\alpha = \frac{-b+\sqrt{b^2-4ac}}{2a} \text{ and } \beta = \frac{-b-\sqrt{b^2-4ac}}{2a}$$

**(i) When $a(\neq 0), b, c \in Q$ and $b^2$ - 4ac is a perfect square**

In this case both the roots are rational and distinct.

**(ii) When $a(\neq 0), b, c \in Q$ and $b^2$ - 4ac is not a perfect square**

In this case both the roots are irrational and distinct. **[See remarks also]**

**Case-2 When $b^2 - 4ac = 0$, (D = 0)**

In this case both the roots are real and equal to $-\frac{b}{2a}$.

**Case-3 When $b^2 - 4ac < 0$, (D < 0)**

In this case $b^2 - 4ac < 0$, then $4ac - b^2 > 0$

$$\therefore \quad \alpha = \frac{-b+\sqrt{-(4ac-b^2)}}{2a} \text{ and } \beta = \frac{-b-\sqrt{(4ac-b^2)}}{2a}$$

or $$\alpha = \frac{-b+i\sqrt{4ac-b^2}}{2a} \text{ and } \beta = \frac{-b-i\sqrt{4ac-b^2}}{2a} \quad [\because \sqrt{-1} = i]$$

i.e. in this case both the root are imaginary and distinct.

## REMARKS:

- ★ If **a,b,c** $\in$ **Q** and **$b^2$ - 4ac** is positive **(D > 0)** but not a perfect square, then the roots are irrational and they always occur in conjugate pairs like $2+\sqrt{3}$ and $2-\sqrt{3}$. However, if **a,b,c** are irrational number and **$b^2$ - 4ac** is positive but not a perfect square, then the roots may not occur in conjugate pairs.
- ★ If **$b^2$ - 4ac** is negative **(D > 0)**, then the roots are complex conjugate of each other. In fact, complex roots of an equation with real coefficients always occur in conjugate pairs like 2 + 3i and 2 - 3i. However, this may not be true in case of equations with complex coefficients. For example, $x^2 - 2ix - 1 = 0$ has both roots equal to i.
- ★ If **a** and **c** are of the same sign and b has a sign opposite to that of a as well as c, then both the roots are positive, the sum as well as the product of roots is positive $(D \geq 0)$.
- ★ If **a,b,** are of the same sign then both the roots are negative, the sum of the roots is negative but the product of roots is positive $(D \geq 0)$.