4.4 METHODS OF SOLVING QUADRATIC EQUATION

(a) By Factorisation:

ALGORITHM:

Step (i) Factorise the constant term of the given quadratic equation.

Step (ii) Express the coefficient of middle term as the sum or difference of the factors obtained in step 1. Clearly, the product of these two factors will be equal to the product of the coefficient of x^2 and constant term.

Step (iii) Split the middle term in two parts obtained in step 2.

Step (iv) Factorise the quadratic equation obtained in step 3.

ILLUSTRATIONS:

Ex.1 Solve the following quadratic equation by factorisation method: $x^2 - 2ax + a^2 - b^2 = 0$.

Sol. Here, Factors of constant term $(a^2 - b^2)$ are $(a - b)$ and $(a + b)$.

Also, Coefficient of the middle term $= - 2a = - [(a - b) + (a + b)]$

$\therefore \quad x^2 - 2ax + a^2 - b^2 = 0$

$\Rightarrow \quad x^2 - \{(a - b) + (a + b)\}x + (a - b)(a + b) = 0$

$\Rightarrow \quad x^2 - (a - b)\,x - (a + b)\,x + (a - b)\,(a + b) =$

$\Rightarrow \quad x\{x - (a - b)\} - (a + b)\,\{x - (a - b)\} = 0$

$\Rightarrow \quad \{x - (a - b)\}\,\{x - (a + b)\} = 0$

$\Rightarrow \quad x - (a - b) = 0$ or, $x - (a + b) = 0$

$\Rightarrow \quad x = a - b$ or $x = a + b$

Ex.2 Solve $64x^2 - 625 = 0$

Sol. We have $64x^2 - 625 = 0$

or $\quad (8x)^2 - (25)^2 = 0$

or $\quad (8x + 25)\,(8x - 25) = 0$

i.e. $\quad 8x + 25 = 0$ o $8x - 25 = 0$.

This gives $x = \frac{25}{8}$ or $\frac{25}{8}$.

Thus, $x = -\frac{25}{8}, \frac{25}{8}$ are solutions of the given equations.

Ex.3 Solve the quadratic equation $16x^2 - 24x = 0$.

Sol. The given equation may be written as $8x(2x - 3) = 0$

This gives $x = 0$ or $x = \frac{3}{2}$.

$x = 0, \frac{3}{2}$, are the required solutions.

Ex.4 Solve :- $25x^2 - 30x + 9 = 0$

Sol. $25x^2 - 30x + 9 = 0$ is equivalent to $(5x)^2 - 2(5x) \times 3 + (3)^2 = 0$

or $(5x - 3)^2 =$

This gives $x = \frac{3}{5}, \frac{3}{5}$ or simply $x = \frac{3}{5}$ as the required solution.

Ex.5 Find the solutions of the quadratic equation $x^2 + 6x + 5 = 0$.

Sol. The quadratic polynomial $x^2 + 6x + 5$ can be facorised as follows :-

$x^2 + 6x + 5 = x^2 + 5x + x + 5$

$= x(x + 5) + 1 (x + 5)$

$= (x + 5) (x + 1)$

Therefore the given quadratic equation becomes $(x + 5) (x + 1) =$

This gives $x = - 5$ or $= - 1$

Therefore, $x = - 1$ are the required solutions of the given equation.

Ex.6 Solve : $\frac{2x}{x-3} + \frac{1}{2x+3} + \frac{3x+9}{(x-3)(2x+3)} = 0.$

Sol. Obviously, the given equation is valid if $x - 3 \neq 0$ and $2x + 3 \neq 0$.

Multiplying throughout by $(x - 3) (2x - 3)$, we get

$2x(2x + 3) + 1(x - 3) + 3x + 9 = 0$

or $4x^2 + 10 + 6 = 0$

or $2x^2 + 5x + 3 = 0$

or $(2x + 3) (x + 1) = 0$

But $2x + 3 \neq 0$, so we get $x + 1 = 0$.

This gives $x = - 1$ as the only solution of the given equation.

(b) By the Method of Completion of Square:

ALGORITHM :

Step-(i) Obtain the quadratic equation. Let the quadratic equation be $ax^2 + bx + c = 0, a \neq 0$.

Step-(ii) Make the coefficient of x^2 unity, if it is not unity. i.e., obtained $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$.

Step-(iii) Shift the constant term $\frac{c}{a}$ on R.H.S. to get $x^2 + \frac{b}{a}x = -\frac{c}{a}$

Step-(iv) Add square of half of the coefficient of x i.e. $\left(\frac{b}{2a}\right)^2$ on both sides to obtain

$$x^2 + 2\left(\frac{b}{2a}\right)x + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$$

Step-(v) Write L.H.S. as the perfect square of a binomial expression and simplify R.H.S. to get

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}.$$

Step-(vi) Take square root of both sides to get $x+\frac{b}{2a}=\pm\sqrt{\frac{b^2-4ac}{4a^2}}$

Step (vii) Obtain the values of x by shifting the constant term $\frac{b}{2a}$ on RHS.

Ex.7 Solve :- $x^2 + 3x + 1 = 0$

Sol. We have

$x^2 + 3x + 1 = 0$

Add and subtract ($\frac{1}{2}$coefficient of x)^2 in L.H.S. and get

$$x^2+3x+1+\left(\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2=0$$

$$\Rightarrow \quad x^2+2\left(\frac{3}{2}\right)x+\left(\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2+1=0$$

$$\Rightarrow \quad \left(x+\frac{3}{2}\right)^2-\frac{5}{4}=0$$

$$\Rightarrow \quad \left(x+\frac{3}{2}\right)^2=\left(\frac{\sqrt{5}}{2}\right)^2$$

$$\Rightarrow \quad x+\frac{3}{2}=\pm\frac{\sqrt{5}}{2}$$

This gives $x=\frac{-\left(3+\sqrt{5}\right)}{2}$ or $x=\frac{-3+\sqrt{5}}{2}$

Therefore $x=-\frac{3+\sqrt{5}}{2},\frac{-3+\sqrt{5}}{2}$ are the solutions of the given equation.

Ex.8 By using the method of completing the square, show that the equation $4a^2 + 3x + 5 = 0$ has no real roots.

Sol. We have, $4x^2 + 3x + 5 = 0$

$$\Rightarrow \quad x^2+\frac{3}{4}x+\frac{5}{4}=0$$

$$\Rightarrow \quad x^2+2\left(\frac{3}{8}x\right)=-\frac{5}{4}$$

$$\Rightarrow \quad x^2+2\left(\frac{3}{8}\right)x+\left(\frac{3}{8}\right)^2=\left(\frac{3}{8}\right)^2-\frac{5}{4}$$

$\Rightarrow \qquad \left(x+\frac{3}{8}\right)^2 = -\frac{71}{64}$

Clearly, RHS is negative

But, $\left(x+\frac{3}{8}\right)^2$ cannot be negative for any real value of x.

Hence, the given equation has no real roots.

(c) By Using Quadratic Formula :

Solve the quadratic equation in general form viz. $ax^2 + bx + c = 0$.

We have, $ax^2 + bx + c = 0$

Step (i) By comparison with general quadratic equation, find the value of a,b and c.

Step (ii) Find the discriminate of the quadratic equation.

$$D = b^2 - 4ac$$

Step (iii) Now find the roots of the equation by given equation

$$x = \frac{-b+\sqrt{D}}{2a}, \frac{-b-\sqrt{D}}{2a}$$

REMARK :

★ If $b^2 - 4ac < 0$ i.e. negative, then $\sqrt{b^2-4ac}$ is not real and therefore, the equation does not have any real roots.

Ex.9 Solve the quadratic equation $x^2 - 7x - 5 = 0$.

Sol. Comparing the given equation with $ax^2 + bx + c = 0$, we find that a = 1, b = - 7 and c = -5.

Therefore, $D = (-7)^2 - 4 \times 1 \times (-5) = 49 + 20 = 69 > 0$

Since D is positive, the equation has two roots given by $\frac{7+\sqrt{69}}{2}, \frac{7-\sqrt{69}}{2}$

$\Rightarrow \qquad x = \frac{7+\sqrt{69}}{2}, \frac{7-\sqrt{69}}{2}$ are the required solutions.

Ex.10 For what value of k, $(4 - k)x^2 + (2k + 4)x + (8k + 1)$ is a perfect square.

Sol. The given equation is a perfect square, if its discriminate is zero i.e. $(2k + 4)^2 - 4(4 - k)(8k + 1) = 0$

$\Rightarrow \qquad 4(k + 2)^2 - 4(4 - k)(8k + 1) = 0 \Rightarrow 4[4(k + 2)^2 - (4 - k)(8k + 1)] = 0$

$\Rightarrow \qquad [(k^2 + 4k + 4) - (-8k^2 + 31k + 4)] = 0 \Rightarrow 9k^2 - 27k = 0$

$\Rightarrow \qquad 9k(k - 3) = 0 \Rightarrow k = 0$ or $k = 3$

Hence, the given equation is a perfect square, if k = 0 or k = 3.

Ex.11 If the roots of the equation $a(b - c)x^2 + b(c - a)x + c(a - b) = 0$ are equal, show that $\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$.

Sol. Since the roots of the given equations are equal, so discriminant will be equal to zero.

$\Rightarrow \quad b^2(c - a)^2 - 4a(b - c) . c(a - b) = 0$

$\Rightarrow \quad b^2(c^2 + a^2 - 2ac) - 4ac(ba - ca - b^2 + bc) = 0,$

$\Rightarrow \quad a^2b^2 + b^2c^2 + 4a^2c^2 + 2b^2ac - 4ac^2bc - 4abc^2 = 0 \Rightarrow (ab + bc - 2ac)^2 = 0$

$\Rightarrow \quad ab + bc - 2ac = 0$

$\Rightarrow \quad ab + bc = 2ac$

$\Rightarrow \quad \frac{1}{c} + \frac{1}{a} = \frac{2}{b}$

$\Rightarrow \quad \frac{2}{b} = \frac{1}{a} + \frac{1}{c}$. **Hence Proved.**

Ex.12 If the roots of the equation $(b - c)x^2 + (c - a) x + (a - b) = 0$ are equal, then prove that $2b = a + c$.

Sol. If the roots of the given equation are equal, then discriminant is zero i.e.

$(c - a)^2 - 4(b - c) (a - b) = 0 \Rightarrow c^2 + a^2 - 2ac + 4b^2 - 4ab + 4ac - 4bc = 0$

$\Rightarrow \quad c^2 + a^2 + 4b^2 + 2ac - 4ab - 4bc = 0$

$\Rightarrow \quad (c + a - 2b)^2 = 0$

$\Rightarrow \quad c + a = 2b$ **Hence Proved.**

Ex.13 If the roots of the equation $x^2 - 8x + a^2 - 6a = 0$ are real and distinct, then find all possible values of a.

Sol. Since the roots of the given equation are real and distinct, we must have $D > 0$

$\Rightarrow \quad 64 - 4 (a^2 - 6a) > 0$

$\Rightarrow \quad 4[16 - a^2 + 6a] > 0$

$\Rightarrow \quad -4(a^2 - 6a - 16) > 0$

$\Rightarrow \quad a^2 - 6a - 16 < 0$

$\Rightarrow \quad (a - 8) (a + 2) < 0$

$\Rightarrow \quad - 2 < a < 8$

Hence, the roots of the given equation are real if **'a'** lies between -2 and 8.