4.5 APPLICATIONS OF QUADRATIC EQUATIONS

ALGORITHM:

The method of problem solving consist of the following three steps :

Step (i) Translating the word problem into symbolic language (mathematical statement) which means identifying relationship existing in the problem and then forming the quadratic equation.

Step (ii) Solving the quadratic equation thus formed.

Step (iii) Interpreting the solution of the equation, which means translating the result of mathematical statement into verbal language.

REMARKS:

- ★ Two consecutive odd natural numbers be $2x - 1, 2x + 1$ where $x \in N$
- ★ Two consecutive even natural numbers be $2x, 2x + 2$ where $x \in N$
- ★ Two consecutive even positive integers be $2x, 2x + 2$ where $x \in Z^+$
- ★ Consecutive multiples of 5 be $5x, 5x + 5, 5x + 10$ .....

Ex.14 The sum of the squares of two consecutive positive integers is 545. Find the integers.

Sol. Let x be one of the positive integers. Then the other integer is $x + 1$, $x \in Z^+$

Since the sum of the squares of the integers is 545, we get

$$x^2 + (x + 1)^2 = 545$$

or $2x^2 + 2x - 544 = 0$

or $x^2 + x - 272 = 0$

$$x^2 + 17x - 16x - 272 = 0$$

or $x(x + 17) - 16(x + 17) = 0$

or $(x - 16)(x + 17) = 0$

Here, $x = 16$ or $x = -17$ But, x is a positive integer. Therefore, reject $x = -17$ and take **$x = 16$.** Hence, two consecutive positive integers are 16 and (16 + 1), i.e., 16 and 17.

Ex.15 The length of a hall is 5 m more than its breath. If the area of the floor of the hall is 84 m^2, what are the length and the breadth of the hall ?

Sol. Let the breadth of the hall be x metres.

Then the length of the ball is (x + 5) metres.

The area of the floor = $x(x + 5)$ m^2

Therefore, $x(x + 5) = 84$

or $x^2 + 5x - 84 = 0$

or $(x + 12)(x - 7) = 0$

This given $x = 7$ or $x = -12$.

Since, the breadth of the hall cannot be negative, we reject $x = -12$ and take $x =$ - only.

Thus, breadth of the hall = 7 metres, and length of the hall = (7 + 5), i.e., 12 metres.

Ex.16 Out of group of swans $\frac{7}{2}$ times the square root of the total number are playing on the shore of a tank. The two remaining ones are playing, in deep water. What is the total number of swans ?

Sol. Let us denote the number of swans by x.

Then, the number of swans playing on the shore of the tank $= \frac{7}{2}\sqrt{x}$.

There are two remaining swans.

Therefore, $x = \frac{7}{2}\sqrt{x} + 2$

or $x - 2 = \frac{7}{2}\sqrt{x}$

or $(x-2)^2 = \left(\frac{7}{2}\right)^2 x$

or $4(x^2 - 4x + 4) = 49x$

or $4x^2 - 65x + 16 = 0$

or $4x^2 - 64x - x + 16 = 0$

or $4x(x - 16) - 1(x - 16) = 0$

or $(x - 16)(4x - 1) = 0$

This gives $x = 16$ or $x = \frac{1}{4}$

We reject $x = \frac{1}{4}$ and take $x = 16$.

Hence, the total number of swans is 16.

Ex.17 The hypotenuse of a right triangle is 25 cm. The difference between the lengths of the other two sides of the triangle is 5 cm. Find the lengths of these sides.

Sol. Let the length of the shorter side b x cm. Then, the length of the longer side = (x + 5) cm.

Since the triangle is right-angled, the sum of the squares of the sides must be equal to the square of the hypotenuse (Pythagoras Theorem).

$x^2 + (x + 5)^2 = 235^2$

or $x^2 + x^2 + 10x + 25 = 625$

or $2x^2 + 10x - 600 = 0$

or $x^2 + 5x - 300 = 0$

or $(x + 20)(x - 15) = 0$

This gives $x = 15$ or $x = -20$

We reject $x = -20$ and take $x = 15$.

Thus, length of shorter side = 15 cm.

Length of longer side = (15 + 5) cm, i.e., 20 cm.

Ex.18 Swati can row her boat at a speed of 5 km/h in still water. If it takes her 1 hour more to row the boat 5.25 km upstream than to return downstream, find the speed of the stream.

Sol. Let the speed of the stream be x km/h

$\therefore$ Speed of the boat in upstream = (5 - x)km/h

Speed of the boat in downstream = (5 + x)km/h

Time, say t_1 (in hours), for going 5.25 km upstream = $\frac{5.25}{5-x}$

Time, say t_2 (in hours), for returning 5.25 km downstream $= \frac{5.25}{5+x}$

Obviously $t_1 > t_2$

Therefore, according to the given condition of the problem,

$t_1 = t_2 + 1$

i.e., $\frac{5.25}{5-x} = \frac{5.25}{5+x} + 1$

or $\frac{21}{4}\left(\frac{1}{5-x} - \frac{1}{5+x}\right) = 1$

or $21\left(\frac{5+x-5+x}{25-x^2}\right) = 4$

or $42x = 100 - 4x^2$

or $4x^2 + 42x - 100 = 0$

or $2x^2 + 21x - 50 = 0$

or $(2x + 25)(x - 2) = 0$

This gives x = 2, since we reject $x = \frac{-25}{2}$.

Thus, the speed of the stream is 2 km/h.

Ex.19 The sum of the square of two positive integers is 208. If the square of the larger number is 18 times the smaller number, find the numbers. **[CBSE - 2007]**

Sol Let x be the smaller number.

Then, square of the larger number will be 18x.

Therefore, $x^2 + 18x = 208$

or $x^2 + 18x - 208 = 0$

or $(x - 8)(x + 26) = 0$

This gives x = 8 or x = - 26

Since the numbers are positive integers, we reject x = - 26 and take x = 8.

Therefore, square of larger number = 18 × 8 = 144.

So, larger number = $\sqrt{144} = 12$

Hence, the larger number is 12 and the smaller is 8.

Ex.20 The sum 'S' of first n natural number is given by the relation $S = \frac{n(n+1)}{2}$. Find n, if the sum is 276.

Sol. We have

$S = \frac{n(n+1)}{2} = 276$

or $n^2 + n - 552 = 0$

This gives $n = \frac{-1+\sqrt{1+2208}}{2}, \frac{-1-\sqrt{1+2208}}{2}$

or $n = \frac{-1+\sqrt{2209}}{2}, \frac{-1-\sqrt{2209}}{2}$

or $n = \frac{-1+47}{2}, \frac{-1-47}{2}$

or n = 23, -24

We reject n = - 24, since -24 is not a natural number.

Therefore, n = 23.