5.2 GENERAL FORM OF AN A.P

If we denote the starting number i.e. the 1st number by **'a'** and a fixed number to the added is **'d'** then **a, a + d, a + 2d, a + 3d, a + 4d, ...** forms an **A.P.**

Ex.2 Find the A.P. whose 1st term is 10 & common difference is 5.

Sol. Given : First term (a) = 10 & Common difference (d) = 5.

∴ A.P. is 10, 15, 20, 25, 30, ...

n^{th} TERM OF AN A.P. :

Let A.P. be a, a + d, a + 2d, a + 3d, ...

Then, First term (a_1) = a + 0.d

Second term (a_2) = a + 1.d

Third term (a_3) = a + 2.d

. .

. .

. .

n^{th} term (a_n) = a + (n - 1) d

∴ **$a_n = a + (n - 1) d$ is called the n^{th} term.**

Ex.3 Determine the A.P. whose their term is 16 and the difference of 5^{th} term from 7^{th} term is 12.

Sol. Given : $a_3 = a + (3 - 1) d = a + 2d = 16$ .....(i)

$a_7 - a_5 = 12$ ....(ii)

$(a + 6d) - (a + 4d) = 12$

$a + 6d - a - 4d = 12$

$2d = 12$

$d = 6$

Put d = 6 in equation (i)

$a = 16 - 12$

$a = 4$

∴ A.P. is 4, 10, 16, 22, 28, ...

Ex.4 Which term of the sequence 72, 70, 68, 66, ... is 40 ?

Sol. Here 1st term x = 72 and common difference d = 70 - 72 = - 2

∴ For finding the value of n

$a_n = a + (n - 1)d$

⇒ $40 = 72 + (n - 1)(-2)$

⇒ $40 - 72 = -2n + 2$

⇒ $-32 = -2n + 2$

⇒ $-34 = -2n$

⇒ $n = 17$

∴ 17^{th} term is 40.

Ex.5 Is 184, a term of the sequence 3,7,11,..... ?

Sol. Here 1st term (a) = 3 and common difference (d) = 7 - 3 = 4

n^{th} term $(a_n) = a + (n - 1)\,d$

$\Rightarrow$ $184 = 3 + (n - 1)\,4$

$\Rightarrow$ $181 = 4n - 4$

$\Rightarrow$ $185 = 4n$

$\Rightarrow$ $n = \dfrac{185}{4}$

Since, n is not a natural number.

$\therefore$ 184 is not a term of the given sequence.

Ex.6 Which term of the sequence 20, $19\frac{1}{2}, 18\frac{1}{2}, 17\frac{3}{4}$ is the 1st negative term.

Sol. Here 1st term (a) = 20, common difference (d) = $19\frac{1}{4} - 20 = -\frac{3}{4}$

Let n^{th} term of the given A.P. be 1st negative term $\therefore$ $a_n < 0$

i.e. $a + (n - 1)\,d < 0$

$\Rightarrow$ $20 + (n - 1)\left(-\dfrac{3}{4}\right) < 0 \Rightarrow \dfrac{83}{4} - \dfrac{3n}{4} < 0$

$\Rightarrow$ $3n > 83 \Rightarrow n > \dfrac{83}{3} \Rightarrow n > 27\dfrac{2}{3}$

Since, 28 is the natural number just greater then $27\frac{2}{3}$.

$\therefore$ 1st negative term is 28th.

Ex.7 If p^{th}, q^{th} and r^{th} term of an A.P. are a,b,c respectively, then show than $a(q - r) + b(- p) + c(p - q) = 0$.

Sol. $a_p = a \Rightarrow A + (p - 1)\,D = a$ ......(1)

$a_q = b \Rightarrow A + (q - 1)\,D = b$ .....(2)

$a_r = c \Rightarrow A + (r + 1)\,D = c$ ......(3)

Now, L.H.S. $= a\,(q - r) + b(r - p) + c\,(p - q)$

$= \{A + (p - 1)D\}\,(q - r) + \{A + (q - 1)D\}\,(r - p) + \{A + (r - 1)D\}\,(p - q)$

$= 0.$ **R.H.S**

Ex.8 If m times the m^{th} term of an A.P. is equal to n times its n^{th} term. Show that the $(m + n)^{th}$ term of the A.P.

Sol. Let A the 1st term and D be the common difference of the given A.P.

Then, $ma_m = na_n$

$\Rightarrow$ $m[A + (m - 1)D] = n[A + (n - 1)D]$

$\Rightarrow$ $A(m - 1) + D[m + n\,(m - n) - (m - n)] = 0$

$\Rightarrow$ $A + (m + n - 1)D = 0$

$\Rightarrow$ $a_{m+n} = 0$

Ex.9 If the p^{th} term of an A.P. is q and the q^{th} term is p, prove that its n^{th} term is $(p + q - n)$.

Sol. $a_p = q \Rightarrow A + (p - 1)\,D = q$ ......(i)

& $a_q = p \Rightarrow A + (q - 1)\,D = p$

Solve (i) & (ii) to get $D = -1$ & $A = p + q - 1$

$\therefore$ $a_n = A + (n - 1)\,D$

$a_n = (p + q - 1) + (n - 1)\,(-1)$

$a_n = p + q - n.$

Ex.10 If the m^{th} term of an A.P. $\frac{1}{n}$ and n^{th} term be $\frac{1}{m}$ then show that its (mn) term is 1.

Sol. $a_m = \frac{1}{n} \Rightarrow A + (m-1)D = \frac{1}{n}$ .....(i)

& $a_m = \frac{1}{m} \Rightarrow A + (n-1)D = \frac{1}{m}$ ....(ii)

By solving (i) & (ii) $D = \frac{1}{mn}$ & $A = \frac{1}{mn}$

$\therefore$ $a_{mn} = A + (mn - 1)\ D = 1.$

m^{th} TERM OF AN A.P. FROM THE END:

Let **'a'** be the 1st term and **'d'** be the common difference of an **A.P.** having **n** terms. **Then m^{th}** term from the end is **$(n - m + 1)^{th}$** term from beginning or **$\{n - (m -)\}^{th}$** term from beginning.

Ex.11 Find 20 **th** term from the end of an A.P. 3,7,11..... 407.

Sol. $407 = 3 + (n - 1)4 \Rightarrow n = 102$

$\therefore 20^{th}$ term from end $\Rightarrow m = 20$

$a_{102-(20-1)} = a_{102-19} = a_{83}$ from the beginning.

$a_{83} = 3 + (83 + 1)4 = 331.$

SELECTION OF TERMS IN AN A.P.:

Sometimes we require certain number of terms in A.P. The following ways of selecting terms are generally very convenient.

No. of Terms	Terms	Common Difference
For 3 terms	a – d, a, a + d	d
For 4 terms	a – 3d, a – d, a + d, a + 3d	2d
For 5 terms	a – 2d, a – d, a, a + d, a + 2d	d
For 6 terms	a – 5d, a – 3d, a – d, a + d, a + 3d, a + 5d	2d

Ex.12 The sum of three number in A.P. is -3 and their product is 8. Find the numbers.

Sol. Three no. 's in A.P. be a - d, a, a + d

$\therefore$ $a - d + a + a + d = -3$

$3a = -3 \Rightarrow a = -1$

& $(a - d)\ a\ (a + d) = 8$

$a(a^2 - d^2) = 8$

$(-1)\ (1 - d^2) = 8$

$1 - d^2 = -8$

$\Rightarrow \quad d^2 = 9$

$\Rightarrow \quad d = \pm\ 3$

If a = 8 & d = 3 numbers are -4, -1, 2.

If a = 8 & d = - numbers are 2, -1, -4.