5.3 SUM OF n TERMS OF AN A.P

Let A.P. be **a, a + d, a + 1d, a + 3d,...... a + (n - 1)d**

Then, $S_n = a + (a + d) + \quad ..... + \{a + (n - 2) d\} + \{a + (n - 1) d\}$ .....(i)

also, $S_n = \{a + (n - 1)d\} + \{a + (n - 2)d\} + ... + (a + d) + a$ .....(ii)

Add (i) & (ii)

$\Rightarrow \quad 2S_n = 2a + (n - 1)d + 2a + (n - 1)d + ... + 2a + (n - 1)d$

$\Rightarrow \quad 2S_n = n [2a + (n - 1) d]$

$\Rightarrow \quad S_n = \frac{n}{2}[2a + (n + 1)d]$

$S_n = \frac{n}{2}[a + a + (n - 1)d] = \frac{n}{2}[a + \ell]$

$\therefore \quad S_n = \frac{n}{2}[a + \ell]$ where ℓ is the last term.

Ex.13 Find the sum of 20 terms of the A.P. 1,4,7,10.....

Sol. a = 1, d = 3

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$S_{20} = \frac{20}{2}[2(1) + (20 - 1)3]$

Ex.14 Find the sum of all three digit natural numbers. Which are divisible by 7.

Sol. 1st no. is 105 and last no. is 994.

Find n

$994 = 105 + (n + 1)7$

$\therefore \quad n = 128$

$\therefore$ Sum, $S_{128} = \frac{128}{2}[105 + 994]$

PROPERTIES OF A.P. :

(A) For any real numbers a and b, the sequence whose nth term is $\mathbf{a_n = an + b}$ is always an **A.P.** with common difference **'a'** (i.e. coefficient of term containing **n**)

(B) If any nth term of sequence is a linear expression in **n** then the given sequence is an **A.P.**

(C) If a constant term is added to or subtracted from each term of an **A.P.** then the resulting sequence is also an **A.P.** with the same common difference.

(D) If each term of a given **A.P.** is multiplied or divided by a non-zero constant **K,** then the resulting sequence is also an **A.P.** with common difference **Kd** or [redacted] respectively. Where **d** is the common difference of the given **A.P.**

(E) In a finite **A.P.** the sum of the terms equidistant from the beginning and end is always same and is equal to the sum of 1st and **last term.**

(F) If three numbers **a,b,c** are in **A.P.,** then **2b = a + c.**

Ex.15 Check whether $a_n = 2n^2 + 1$ is an **A.p.** or not.

Sol. $a_n = 2n^2 + 1$

Then $a_{n+1} = 2(n + 1)^2 + 1$

$\therefore$ $a_{n+1} - a_n = 2(n^2 + 2n + 1) + 1 - 2n^2 - 1$

$= 2n^2 + 4n + 2 + 1 - 2n^2 - 1$

$= 4n + 2$, which is not constant

$\therefore$ The above sequence is not an A.P.