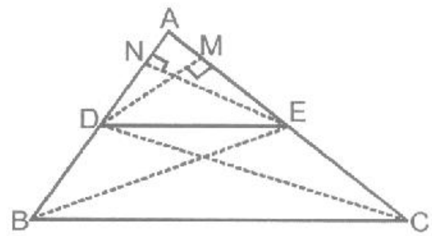
6.2 THALES THEOREM
(BASIC PROPROTIONALITY THEOREM)

Statement: If a line is drawn parallel to one side of a triangle to intersect the other sides in distinct points, then the other two sides are divided in the same ratio.



Given: A triangle ABC in which a line parallel to side BC intersects other two sides AB and AC at D and E respectively.

To Prove : $\frac{AD}{DB} = \frac{AE}{EC}$

Construction : Join BE and CD and draw DM ⊥ AC and EN ⊥ AB.

Proof : Area of ΔADE (= $\frac{1}{2}$ base × height) = $\frac{1}{2}$ AD × EN.

Area of ΔADE is denoted as are (ADE)

So, ar(ADE) = $\frac{1}{2}$ DB × EN

And ar(BDE) = $\frac{1}{2}$ DB × EN ,

Therefore, $$\frac{ar(ADE)}{ar(BDE)} = \frac{\frac{1}{2}AD \times EN}{\frac{1}{2}DB \times EN} = \frac{AD}{DB} \quad ....(i)$$

Similarly, ar(ADE = $\frac{1}{2}$ AE × DM and ar(DEC = $\frac{1}{2}$ EC × DM.

And $$\frac{ar(ADE)}{ar(DEC)} = \frac{\frac{1}{2}AE \times DM}{\frac{1}{2}EC \times DM} = \frac{AE}{EC} \quad ......(ii)$$

Note that Δ BDE and Δ DEC are on the same base DE and between the two parallel lines BC and DE.

So, ar(BDE) = ar(DEC) .....(iii)

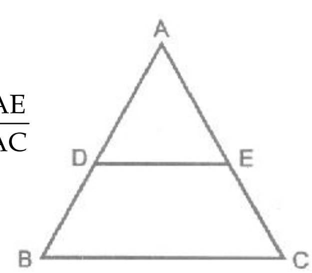
Therefore, from (i), (ii) and (iii), we have :

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Hence Proved.

Corollary :

If in a ΔABC, a line DE | | BC, intersects AB in D and AC in E, then



(i) $\frac{DB}{AD} = \frac{EC}{AE}$ (ii) $\frac{AB}{AD} = \frac{AC}{AE}$ (ii) $\frac{AD}{AB} = \frac{AE}{AC}$

(iv) $\frac{AB}{DB} = \frac{AC}{EC}$ (v) $\frac{DB}{AB} = \frac{EC}{AC}$

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Converse of Basic Proportionality Theorem :

If a line divides any two sides of a triangle in the same ratio, then the line must be parallel to the third side.

Some Important Results and Theorems :

(i) The internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

(ii) In a triangle **ABC**, if **D** is a point on **BC** such that **D** divides **BC** in the ratio **AB : AC,** then **AD** is the bisector of Ď**A.**

(iii) The external bisector of an angle of a triangle divides the opposite sides externally in the ratio of the sides containing the angle.

(iv) The line drawn from the mid-point of one side of a triangle parallel to another side bisects the third side.

(v) The line joining the mid-points of two sides of a triangle is parallel to the third side.

(vi) The diagonals of a trapezium divide each other proportionally.

(vii) If the diagonals of a quadrilateral divide each other proportionally, then it is a trapezium.

(viii) Any line parallel to the parallel sides of a trapezium divides the non-parallel sides proportionally.

(ix) If three or more parallel lines are intersected by two transversal, then the intercepts made by them on the transversal are proportional.

ILLUSTRATIONS:

Ex.1 In a ΔABC, D and E are points on the sides AB and AC respectively such that DE || BC. If AD = 4x - 3, AE = 8x - 7, BD = 3x - 1 and CE = 5x - 3, find the value of x.

[CBSE - 2006]

Sol. In ΔABC, we have

DE||BC

∴ $\frac{AD}{DB} = \frac{AE}{EC}$ [By Basic Proportionality Theorem]

⇒ $\frac{4x-3}{3x-1} = \frac{8x-7}{5x-3}$

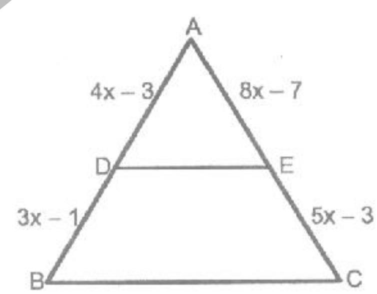
⇒ $20x^2 - 15x - 12x + 9 = 24x^2 - 21x - 8x + 7$

⇒ $20x^2 - 27x + 9 = 24x^2 - 29x + 7$

⇒ $4x^2 - 2x - 2 = 0$

⇒ $2x^2 - x - 1 = 0$

⇒ $(2x + 1)(x - 1) = 0$



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$\Rightarrow \qquad x = 1 \text{ or } x = -\frac{1}{2}$

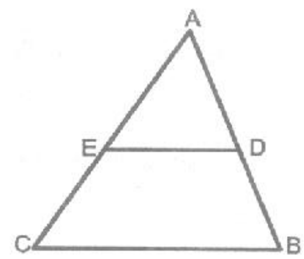
So, the required value of x is 1.

[$x = -\frac{1}{2}$ is neglected as length can not be negative].

Ex.2 D and E are respectively the points on the sides AB and AC of a ΔABC such that AB = 12 cm, AD = 8 cm, AE = 12 cm and AC = 18 cm, show that DE | | BC.

Sol. We have,

AB = 12 cm, AC = 18 m, AD = 8 cm and AE = 12 cm.



$\therefore \qquad$ BD = AB - AD = (12 - 8) cm = 4 cm

CE = AC - AE = (18 12) cm = 6 cm

Now, $\frac{AD}{BC} = \frac{8}{4} = \frac{2}{1}$

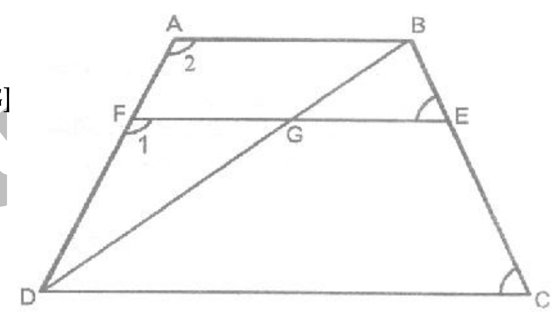
And, $\frac{AE}{CE} = \frac{12}{6} = \frac{2}{1}$

$\Rightarrow \qquad \frac{AD}{BD} = \frac{AE}{CE}$

Thus, DE divides sides AB and AC of ΔABC in the same ratio. Therefore, by the conserve of basic proportionality theorem we have DE| |BC.

Ex.3 In a trapezium ABCD AB| |DC and DC = 2AB. EF drawn parallel to AB cuts AD in F and BC in E such that $\frac{BE}{EC} = \frac{3}{4}$. Diagonal DB intersects EF at G. Prove that 7FE = 10AB.

Sol. In ΔDFG and ΔDAB,



$\angle 1 = \angle 2 \qquad$ [Corresponding ∠s ∴ AB | | FG]

∠FDG = ∠ADB [Common]

$\therefore \qquad$ ΔDFG ~ ΔDAB [By AA rule of similarity]

$\therefore \qquad \frac{DF}{DA} = \frac{FG}{AB} \qquad$ .....(i)

Again in trapezium ABCD

EF| |AB| |DC

$\therefore \qquad \frac{AF}{DF} = \frac{BE}{EC}$

$\Rightarrow \qquad \frac{AF}{DF} = \frac{3}{4} \qquad \left[\because \frac{BE}{EC} = \frac{3}{4} (\text{given})\right]$

$\Rightarrow \qquad \frac{AF}{DF} = 1 = \frac{3}{4} + 1$

$\Rightarrow \qquad \frac{AF + DF}{DF} = \frac{7}{4}$

$\Rightarrow \quad \dfrac{AD}{DF} = \dfrac{7}{4}$

$\Rightarrow \quad \dfrac{DF}{AD} = \dfrac{4}{7}$ …….(ii)

From (i) and (ii), we get

$\dfrac{FG}{AB} = \dfrac{4}{7}$ i.e. FG = $\dfrac{4}{7}AB$ ......(iii)

In ΔBEG and ΔBCD, we have

$\angle BEG = \angle BCD$ [Corresponding angle $\therefore$ EG||CD]

$\angle GBE = \angle DBC$ [Common]

$\therefore$ ΔBEG ~ ΔBCD [By AA rule of similarity]

$\therefore \quad \dfrac{BE}{BC} = \dfrac{EG}{CD}$

$\therefore \quad \dfrac{3}{7} = \dfrac{EG}{CD} \quad \left[\because \dfrac{BE}{EG} = \dfrac{3}{7} \text{ i.e.. } \dfrac{EC}{BE} = \dfrac{4}{3} \Rightarrow \dfrac{EC+BE}{BE} = \dfrac{4+3}{3}\right] \Rightarrow \dfrac{BC}{BE} = \dfrac{7}{3}$

$\therefore \quad EG = \dfrac{3}{7}CD = \dfrac{3}{7}(2\,AB) \quad [\because CD = 2AB \text{ (given)}]$

$\therefore \quad EG = \dfrac{6}{7}AB$ .....(iv)

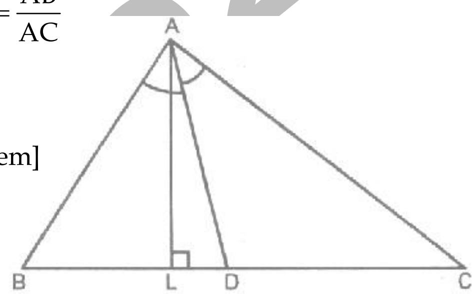
Adding (iii) and (iv), we get

$FG + EG = \dfrac{4}{7}AB + \dfrac{6}{7}AB = \dfrac{10}{7}AB$

$\Rightarrow \quad EF = \dfrac{10}{7}AB$ i.e., $7EF = 10AB$. **Hence proved.**

Ex.4 In ΔABC, if AD is the bisector of ∠A, prove that $\dfrac{\text{Area }(\Delta ABD)}{\text{Area }(\Delta ACD)} = \dfrac{AB}{AC}$

Sol. In ΔABC, AD is the bisector of ∠A.



$\therefore \quad \dfrac{AB}{AC} = \dfrac{BD}{DC}$ ....(i) [By internal bisector theorem]

From A draw AL ⊥ BC

$\therefore \quad \dfrac{\text{Area }(\Delta ABD)}{\text{Area }(\Delta ACD)} = \dfrac{\frac{1}{2}BD.AL}{\frac{1}{2}DC.AL} = \dfrac{BD}{DC} = \dfrac{AB}{AC}$ [From (i)] **Hence Proved.**

Ex.5 $\angle BAC = 90^0$, AD is its bisector. IF DE ⊥ AC, prove that DE × (AB + AB) = AB × AC.

Sol. It is given that AD is the bisector of ∠A of ΔABC.

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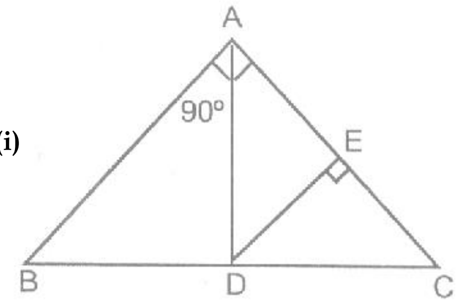
$\therefore \quad \frac{AB}{AC} = \frac{BD}{DC}$

$\Rightarrow \quad \frac{AB}{AC} + 1 = \frac{BD}{DC} + 1$ [Adding 1 on both sides]

Ĝ $\quad \frac{AB + AC}{AC} = \frac{BD + DC}{DC}$



Ĝ $\quad \frac{AB + AC}{AC} = \frac{BC}{DC}$ **....(i)**

In Δ's CDE and CBA, we have

$\angle DCE = \angle BCA$ [Common]

$\angle DEC = \angle BAC$ [Each equal to 90^0]

So, by AA-criterion of similarity

$\Delta\ CDE \sim \Delta\ CBA$

$\Rightarrow \quad \frac{CD}{CB} = \frac{DE}{BA}$

$\Rightarrow \quad \frac{AB}{DE} = \frac{BC}{DC}$ ....(ii)

From (i) and (ii), we have

$\Rightarrow \quad \frac{AB + AC}{AC} = \frac{AB}{DE}$

$\Rightarrow \quad DE \times (AB + AC) = AB \times AC.$

Ex.6 In the given figure, PA, QB and RC are each perpendicular to AC. Prove that $\frac{1}{x} + \frac{1}{z} = \frac{1}{y}$

Sol. In ΔPAC, we have BQ | | AP

$\Rightarrow \quad \frac{BQ}{AP} = \frac{CB}{CA}$ [$\therefore$ ΔCBQ ~ ΔCAP]

$\Rightarrow \quad \frac{y}{x} = \frac{CB}{CA}$ ...(i)

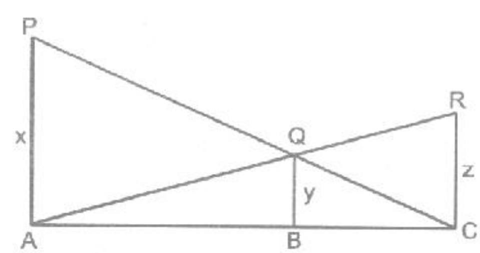
In ΔACR, we have BQ | | CR

$\Rightarrow \quad \frac{BQ}{CR} = \frac{AB}{AC}$ [$\therefore$ Δ ABQ ~ ΔACR]

$\Rightarrow \quad \frac{y}{z} = \frac{AB}{AC}$ ...(ii)

Adding (i) and (ii), we get

$\frac{y}{x} + \frac{y}{z} = \frac{CB}{AC} + \frac{AB}{AC}$



$\Rightarrow \quad \frac{y}{x} + \frac{y}{z} = \frac{AB + BC}{AC} \quad \Rightarrow \quad \frac{y}{x} + \frac{y}{z} = \frac{AC}{AC}$

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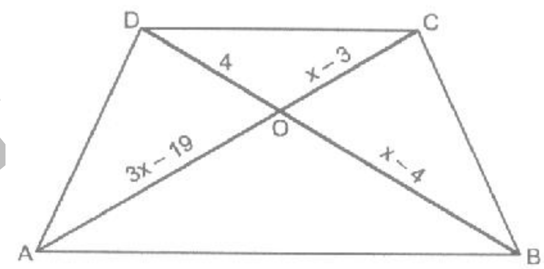
$\Rightarrow \quad \frac{y}{x}+\frac{y}{z}=1 \qquad \Rightarrow \quad \frac{1}{x}+\frac{1}{z}=\frac{1}{y}$ **Hence Proved.**

Ex.7 In the given figure, AB||CD. Find the value of x.

Sol. Since the diagonals of a trapezium divide each other proportionally.



$\therefore \quad \frac{AO}{OC}=\frac{BO}{OD}$

$\Rightarrow \quad \frac{3x-19}{x-3}=\frac{x-4}{4}$

$\Rightarrow \quad 12x - 76 = x^2 - 4x - 3x + 12$

$\Rightarrow \quad x^2 - 19x + 88 = 0$

$\Rightarrow \quad x^2 - 11x - 8x + 88 = 0$

$\Rightarrow \quad (x - 8)(x - 11) = 0$

$\Rightarrow \quad x = 8$ or $x = 11$.

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