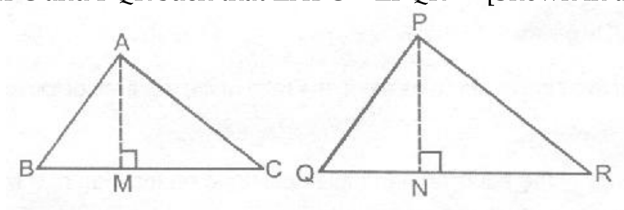
6.3 AREAS OF SIMILAR TRIANGLS

Statement: The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Given: Two triangles ABC and PQR such that $\Delta ABC \sim \Delta PQR$ [Shown in the figure]



To Prove : $\frac{ar(ABC)}{ar(PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{CA}{RP}\right)^2$

Construction : Draw altitudes AM and PN of the triangle ABC an PQR.

Proof : $ar(ABC) = \frac{1}{2} BC \times AM$

And $ar(PQT) = \frac{1}{2} QR \times PN$

So, $\frac{ar(ABC)}{ar(PQR)} = \frac{\frac{1}{2} BC \times AM}{\frac{1}{2} QR \times PN} = \frac{BC \times AM}{QR \times PN}$ ....(i)

Now, in Δ ABM and Δ PQN,

And $\angle B = \angle Q$ [As Δ ABC ~ ΔPQR]

$\angle M = \angle N$ [90^0 each]

So, $\Delta ABM \sim \Delta PQN$ [AA similarity criterion]

Therefore, $\frac{AM}{PN} = \frac{AB}{PQ}$ ....(ii)

Also, $\Delta ABC \sim \Delta PQR$ [Given]

So, $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$ .....(iii)

Therefore, $\frac{ar(ABC)}{ar(PQR)} = \frac{BC}{QR} \times \frac{AB}{PQ}$ [From (i) and (ii)]

$= \frac{AB}{PQ} \times \frac{AB}{PQ}$ [From (iii)]

$= \left(\frac{AB}{PQ}\right)^2$

Now using (iii), we get

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{CA}{RP}\right)^2$$

Properties of Areas of Similar Triangles :

(i) The areas of two similar triangles are in the ratio of the squares of corresponding altitudes.

(ii) The areas of two similar triangles are in the ratio of the squares of the corresponding medians.

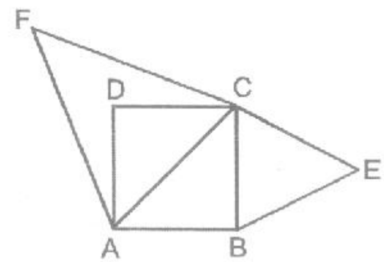
(iii) The area of two similar triangles are in the ratio of the squares of the corresponding angle bisector segments.

Ex.8 Prove that the area of the equilateral triangle described on the side of a square is half the area of the equilateral triangle described on its diagonals.

Sol. **Given:** A square ABCD. Equilateral triangles ΔBCE and ΔACF have been described on side BC and diagonals AC respectively.

To prove: Area (ΔBCE) = $\frac{1}{2}$. Area (ΔACF)

Proof: Since ΔBCE and ΔACF are equilateral. Therefore, they are equiangular (each angle being equal to 60^0) and hence ΔBCE ~ ΔACF.



$$\Rightarrow \quad \frac{Area(\Delta BCE)}{Area(\Delta ACF)} = \frac{BC^2}{AC^2}$$

$$\Rightarrow \quad \frac{Area(\Delta BCE)}{Area(\Delta ACF)} = \frac{BC^2}{(\sqrt{2}BC)^2} = \frac{1}{2}$$

$$\left[\begin{array}{l} \because \text{ABCD is a square} \\ \because \text{Diagonal} = \sqrt{2}(\text{side}) \\ \Rightarrow AC = \sqrt{2}BC \end{array}\right]$$

$$\Rightarrow \quad \frac{Area(\Delta BCE)}{Area(\Delta ACF)} = \frac{1}{2}$$

Hence Proved.