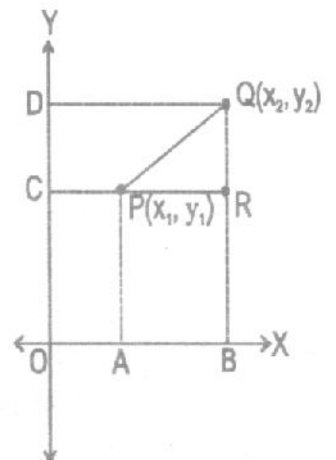
7.2 DISTACE BETWEEN TWO POINTS

Let two points be **P (x_1, y_1)** and **Q(x_2, y_2)**

Take two mutually perpendicular lines as the coordinate axis with **O** as origin. Mark the points **P(x_1, y_1)** and **Q (x_2, y_2)**. Draw lines **PA, QB** perpendicular to **X-axis** from the points **P** and **Q,** which meet the **X-axis** in points A and B, respectively.



Draw lines PC and QD perpendicular to **Y-axis,** which meet the **Y-axis** in C and D, respectively. Produce CP to meet BQ in R. Now

OA = abscissa of P = x_1

Similarly, OB = x_2, OC = y_1 and OD = y_2

Therefore, we have

PR = AB = OB - OA = $x_2 - x_1$

Similarly, QR = QB - RB = QB - PA = $y_2 - y_1$

Now, using Pythagoras Theorem, in right angled triangle PRQ, we have

$$PQ^2 = Pr^2 + RQ^2$$

or $$PQ^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

Since the distance or length of the line-segment PQ is always non-negative, on taking the positive square root, we get the distance as

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

This result is known as **distance formula.**

Corollary : The distance of a point **P(x_1, y_1)** from the origin **(0,0)** is given by

$$OP = \sqrt{x_1^2 + y_1^2}$$

Some useful points :

1.In questions relating to geometrical figures, take the given vertices in the given order and proceed as indicated.

(i) For an **isosceles triangle -** We have to prove that at least two sides are equal.

(ii) For an **equilateral triangle -** We have to prove that three sides are equal.

(iii) For a **right -angled triangle** - We have to prove that the sum of the squares of two sides is equal to the square of the third side.

(iv) for a **square** - We have to prove that the four sides are equal, two diagonals are equal.

(v) For a **rhombus** - We have to prove that four sides are equal (and there is no need to establish that two diagonals are unequal as the square is also a rhombus).

(vi) For a **rectangle** - We have to prove that the opposite sides are equal and two diagonals are equal.

(vii) For a **Parallelogram** - We have to prove that the opposite sides are equal (and there is no need to establish that two diagonals are unequal sat the rectangle is also a parallelogram).

2. for three points to be **collinear -** We have to prove that the sum of the distances between two pairs of points is equal to the third pair of points.

Ex.1 Find the distance between the points (8 , -2) and (3, -6).

Sol. Let the points (8, -2) and (3, -6) be denoted by P and Q, respectively.

Then, by distance formula, we obtain the distance PQ as

$PQ = \sqrt{(3-8)^2 + (-6+2)^2}$

$= \sqrt{(-5)^2 + (-4)^2} = \sqrt{41}$ unit

Ex.2 Prove that the points $(1,-1), \left(-\frac{1}{2}, \frac{1}{2}\right)$ and (1, 2) are the vertices of an isosceles triangle.

Sol. Let the point (1, -1), $\left(-\frac{1}{2}, \frac{1}{2}\right)$ and (1, 2) be denoted by P, Q and R, respectively. Now

$PQ = \sqrt{\left(-\frac{1}{2}-\right)^2 + \left(\frac{1}{2}+1\right)^2} = \sqrt{\frac{18}{4}} = \frac{3}{2}\sqrt{2}$

$QR = \sqrt{\left(1+\frac{1}{2}\right)^2 + \left(2-\frac{1}{2}\right)^2} = \sqrt{\frac{18}{4}} = \frac{3}{2}\sqrt{2}$

$PR = \sqrt{(1-1)^2 + (2+1)^2} = \sqrt{9} = 3$

From the above, we see that PQ = QR

$\therefore$ The triangle is isosceles.

Ex.3 Using distance formula, show that the points (-3, 2), (1, -2) and (9, -10) are collinear.

Sol. Let the given points (-3, 2), (1, -2) and (9, -10) be denoted by A, B and C, respectively. Points A, B and C will be collinear, if the sum of the lengths of two line-segments is equal to the third.

Now, $AB = \sqrt{(1+3)^2 + (-2-2)^2} = \sqrt{16+16} = 4\sqrt{2}$

$BC = \sqrt{(9-1)^2 + (-10+2)^2} = \sqrt{64+64} = 8\sqrt{2}$

$AC = \sqrt{(9+3)^2 + (-10-2)^2} = \sqrt{144+144} = 12\sqrt{2}$

Since, AB + BC = $4\sqrt{2} + 8\sqrt{2} = 12\sqrt{2} = AC$, the, points A, B and C are collinear.

Ex.4 Find a point on the X-axis which is equidistant from the points (5, 4) and (-2, 3).

Sol. Since the required point (say P) is on the X-axis, its ordinate will be zero. Let the abscissa of the point be x.

Therefore, coordinates of the point P are (x, 0).

Let A and B denote the points (5, 4) and (-2, 3), respectively.

Since we are given that AP = BP, we have

$AP^2 = BP^2$

i.e., $(x - 5)^2 + (0 - 4)^2 = (x + 2)^2 + (0 - 3)^2$

or $x^2 + 25 - 10x + 16 = x^2 + 4 + 4x + 9$

or $-14x = -28$

or $x = 2$

Thus, the required point is (2, 0).

Ex.5 The vertices of a triangle are (-2, 0) , (2, 3) and (1, -3). Is the triangle equilateral, isosceles or scalene?

Sol. Let the points (-2, 0), (2, 3) and (1, -3) be denoted by A, B and C respectively. Then,

$AB = \sqrt{(2+2)^2 + (3-0)^2} = 5$

$BC = \sqrt{(1-2)^2 + (-3-3)^2} = \sqrt{37}$

and $AC = \sqrt{(1+2)^2 + (-0-0)^2} = 3\sqrt{2}$

Clearly, $AB \neq BC \neq AC$.

Therefore, ABC is a scalene triangle.

Ex.6 The length of a line-segments is 10. If one end is at (2, -3) and the abscissa of the second end is 10, show that its ordinate is either 3 or -9.

Sol. Let (2, -3) be the point A. let the ordinate of the second end B be y. Then its coordinates will be (10, y).

$\therefore$ $AB = \sqrt{(10-2)^2 + (y+3)^2} = 10$ (Given)

or $64 + 9 + y^2 + 6y = 100$

or $y^2 + 6y + 73 - 100 = 0$

or $y^2 + 6y - 27 = 0$

or $(y + 9)(y - 3) = 0$

Therefore, $y = 9$ or $y = 3$.

Ex.7 Show that the points (-2, 5), (3, -4) and (7, 10) are the vertices of a right triangle.

Sol. Let the three points be A(-2, 5), B(3, - 4) and C(7, 10).

Then $AB^2 = (3 + 2)^2 + (-4 - 5)^2 = 106$

$BC^2 = (7 - 3)^2 + (10 + 4)^2 = 212$

$AC^2 = (7 + 2)^2 + (10 - 5)^2 = 106$

We see that

$BC^2 = AB^21 + AC^2$

212 = 106 + 106

212 = 212

$\therefore$ $\angle A = 90^0$

Thus, ABC is a right triangle, right angled at A.

Ex.8 If the distance of P (x, y) from A (5, 1) and B(-1, 5) are equal, prove that 3x = 2y.

Sol. P(x, y), A (5, 1) and B (-1, 5) are the given points.

AP = BP (Given)

$\therefore$ $AP^2 = BP^2$

or $AP^2 - BP^2 = 0$

or $\{(x - 5)^2 + (y - 1)\}^2 - \{(x + 1)^2 + (y - 5)^2\} = 0$

or $x^2 + 25 - 10x + y^2 + 1 - 2y - x^2 - 1 - 2x - y^2 - 25 + 10y = 0$

or $-12x + 8y = 0$

or $3xx = 2y$.