# 7.3 SECTION FORMULAE

**(a) Formula for Internal Division:**

The coordinates of the pint which divided the line segment joining the pints $(x_1, y_1)$ and $x_2, y_2)$ internally in the ratio **m : n** are given by $x = \frac{mx_2 + nx_1}{m+n}, y = \frac{my_2 + my_1}{m+n}$

**Proof :**

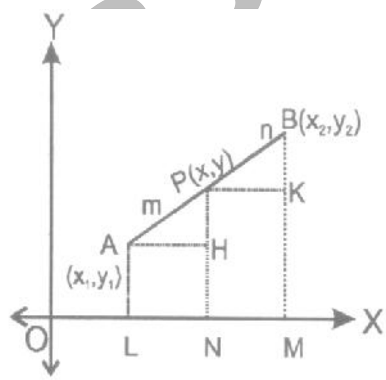
Let O be the origin and let OX and OY be the X-axis and Y-axis respectively. Let **A($x_1$, $y_1$)** and **B($x_2$, $y_2$)** bet the given points. Let **(x, y)** be the coordinates of the point **p** which divides **AB** internally in the ratio **m : n** Draw $AL \perp OX$, $BM \perp OX$, $PN \perp Ox$. Also, draw **AH** and **PK** perpendicular from **A** and **P** on **PN** and **BM** respectively. Then

$OL = x_1$, $ON = x$, $OM = x_2$, $AL = y_1$, $PN = y$ and $BM = y_2$.

$\therefore$ $AH = LN = ON - OL = x - x_1$, $PH = PH - HN$

$= PN - AL = y - y_1$, $PK = NM = OM - ON = x_2 - x$

and $BK = BM - MK = BM - PN = y_2 - y$.




Clearly, $\Delta AHP$ and $\Delta PKB$ are similar.

$\therefore \quad \frac{AP}{BP} = \frac{AH}{PK} = \frac{PH}{BK}$

$\Rightarrow \quad \frac{m}{n} = \frac{x - x_1}{x_2 - x} = \frac{y - y_1}{y_2 - y}$

Now, $\frac{m}{n} = \frac{x - x_1}{x_2 - x}$

$\Rightarrow \quad mx_2 - mx = nx - nx_1 \qquad \Rightarrow \quad mx + nx = mx_2 + nx_1$

$\Rightarrow \quad x = \frac{mx_2 + nx_1}{m+n}$

and $\frac{m}{n} = \frac{y - y_1}{y_2 - y}$

$\Rightarrow \quad my_2 - my = ny - ny_1 \qquad \Rightarrow \quad my + ny = my_2 + ny_1$

$\Rightarrow \quad y = \frac{my_2 + ny_1}{m+n}$

Thus, the coordinates of P are $\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$

**REMARKS**

If **P** is the mid-point of **AB,** then it divides **AB** in the ratio **1 : 1,** so its coordinates are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

## (b) Formula for External Division:

The coordinates of the points which divides the line segment joining the points **($x_1$, $y_1$)** and **($x_2$, $y_2$)** externally in the ratio **m : n** are given by

$x = \frac{mx_2 - nx_1}{m-n}, y = \frac{my_2 - ny_1}{m-n}$

**Ex.9** Find the coordinates of the point which divides the line segment joining the points (6, 3) and (-4, 5) in the ratio 3 : 2 (i) internally (ii) externally.

**Sol.** Let P(x, y) be the required point.

(i) For internal division, we have

$$x = \frac{3x-4+2\times 6}{3+2}$$

and $$y = \frac{3\times 5+2\times 3}{3+2}$$

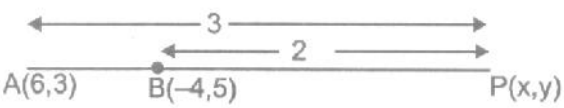

$\Rightarrow$ x = 0 and y = $\frac{21}{5}$

So the coordinates of P are (0, 21/5)

(ii) For external division, we have

$$x = \frac{3x-4-2\times 6}{3-2}$$

any $$y = \frac{3\times 5-2\times 3}{3-2}$$




$\Rightarrow$ x = - 24 and y = 9

So the coordinates of P are (-24, 9).

**Ex.10** In which ratio does the point (-1, -1) divides the line segment joining the pints (4, 4) and (7, 7) ?

**Sol.** Suppose the point C(-1, -1) divides the line joining the points A(4, 4) and B(7, 7) in the ratio k : 1 Then, the coordinates of C are $\left(\frac{7k+4}{k+1}, \frac{7k+4}{k+1}\right)$

But, we are given that the coordinates of the points C are (-1, -1).

$\therefore$ $\frac{7k+4}{k+1} = -1 \Rightarrow k = -\frac{5}{8}$

Thus, C divides AB externally in the ratio 5 : 8.

**Ex.11** In what ratio does the X-axis divide the line segment joining the points (2, -3) and (5, 6) ?

**Sol.** Let the required ratio be k : 1. Then the coordinates of the point of division are $\left(\frac{5\lambda+2}{k+1}, \frac{6\lambda-3}{k+1}\right)$. But, it is a point on X-axis on which y-coordinate of every point is zero.

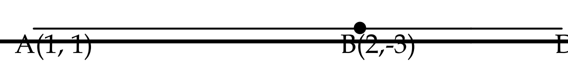
$\therefore$ $\frac{6\lambda-3}{k+1} = 0$

$\Rightarrow$ $k = \frac{1}{2}$

Thus, the required ratio is $\frac{1}{2}$ : 1 or 1 : 2.

**Ex.12** A (1, 1) and B(2, -3) are two points and D is a point on AB produced such that AD = 3 AB. Find the coordinates of D.

**Sol.** We have, AD = 3AB. Therefore, BD = 2AB. Thus D divides AB externally in the ratio AD : BD = 3 : 2 Hence, the coordinates of D are



$\therefore \quad \left(\frac{3\times 2-2\times 1}{3-2}, \frac{3x-3-2\times 1}{3-2}\right)$

$= \quad (4, -11).$

**Ex.13** Determine the ratio in which the line 3x + y - 9 = 0 divides the segment joining the pints (1, 3) and (2, 7).

**Sol.** Suppose the line 3x + y - 9 = 0 divides the line segment joining A(1, 3) and B(2, 7) in the ratio k : 1 at point C. The, the coordinates of C are $\left(\frac{2k+1}{k+1}, \frac{7k+3}{k+1}\right)$ But, C lies on 3x + y - 9 = 0, therefore

$$3\left(\frac{2k+1}{k+1}\right)+\frac{7k+3}{k+1}-9=0$$

$\Rightarrow \quad 6k + 3 + 7k + 3 - 9k - 9 = 0$

$\Rightarrow \quad k=\frac{3}{4}$

So, the required ratio is 3 : 4 internally.

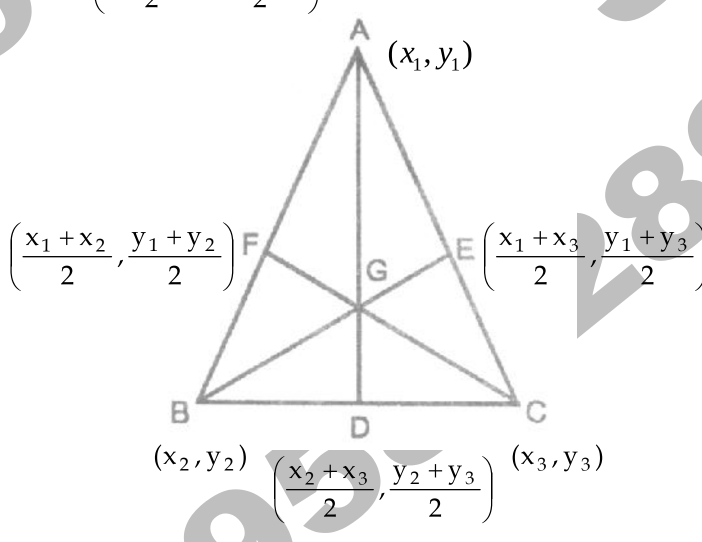
**CENTROID OF A TRIANGLE:**

Prove that the coordinates of the triangle whose vertices are $(x_1, y_1)$, $(x_2, y_2)$ and $(y_3, y_3)$ are $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$. Also, deduce that the medians of a triangle are concurrent.

**Proof :**

Let **A($x_1$, $y_1$, B($x_2$, $y_2$)** and **C($x_3$, $y_3$)** be the vertices of a**ABC** whose medians are AD, BE and CF respectively. So. D,E and F are respectively the mid-points of BC, CA and AB.

Coordinates of **D** are $\left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$. Coordinates of a point dividing AD in the ratio **2 : 1** are



$$\left(\frac{1.x_1+2\left(\frac{x_2+x_3}{2}\right)}{1+2}, \frac{1.y_1+\left(\frac{y_2+y_3}{2}\right)}{1+2}\right)=\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

The coordinates of E are $\left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}\right)$. The coordinates of a point dividing BE in the ratio 2 : 1 are

$$\left(\frac{1.x_2+\frac{2(x_1+x_3)}{2}}{1+2}, \frac{1.y_2+\frac{2(y_1+y_3)}{2}}{1+2}\right)=\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

Similarly the coordinates of a point dividing CF in the ratio 2 : 1 are $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

Thus, the point having coordinates $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$ is common to AD, BE and CF and divides them in the ratio 1 : 2.

Hence, medians of a triangle are concurrent and the coordinates of the centroid are

$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$.