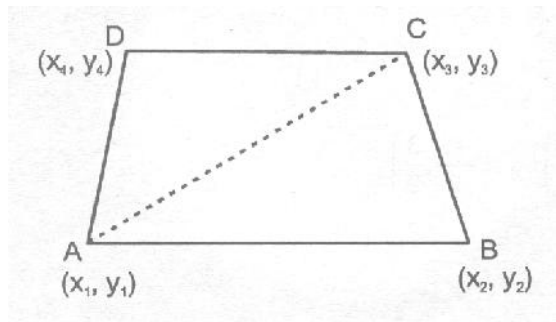


7.5 AREA OF QUADRILATERAL

Let the vertices of Quadrilateral ABCD are $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ and $D(x_4, y_4)$

So, Area of quadrilateral ABCD = Area of $\triangle ABC$ + Area of $\triangle ACD$



Ex.14 The vertices of $\triangle ABC$ are $(-2, 1)$, $(5, 4)$ and $(2, -3)$ respectively. Find the area of triangle.

Sol. $A(-2, 1)$, $B(5, 4)$ and $C(2, -3)$ be the vertices of triangle.

So, $x_1 = -2$, $y_1 = 1$; $x_2 = 5$, $y_2 = 4$; $x_3 = 2$, $y_3 = -3$

$$\begin{aligned} \text{Area of } \triangle ABC &= \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] \\ &= \frac{1}{2} [(-2)(4 - (-3)) + 5(-3 - 1) + 2(1 - 4)] \\ &= \frac{1}{2} [-14 + (-20) + (-6)] \\ &= \frac{1}{2} |-40| \\ &= 20 \text{ Sq. unit.} \end{aligned}$$

Ex.15 The area of a triangle is 5. Two of its vertices are $(2, 1)$ and $(3, -2)$. The third vertex lies on $y = x + 3$. Find the third vertex.

Sol. Let the third vertex be (x_3, y_3) area of triangle

$$= \frac{1}{2} [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

As $x_1 = 2$, $y_1 = 1$; $x_2 = 3$, $y_2 = -2$; Area of $\triangle = 5$ sq. unit

$$\Rightarrow 5 = \frac{1}{2} [2(-2 - y_3) + 3(y_3 - 1) + x_3(1 + 2)]$$

$$\Rightarrow 10 = |3x_3 + y_3 - 7|$$

$$\Rightarrow 3x_3 + y_3 - 7 = \pm 10$$

Taking positive sign

$$3x_3 + y_3 - 7 = 10 \Rightarrow 3x_3 + y_3 = 17 \quad \dots(i)$$

Taking negative sign

$$\Rightarrow 3x_3 + y_3 - 7 = -10$$

$$\Rightarrow 3x_3 + y_3 = -3 \quad \dots(ii)$$

Given that (x_3, y_3) lies on $y = x + 3$

$$\text{So, } -x_3 + y_3 = 3 \quad \dots(iii)$$

Solving eq. (i) & (iii)

$$x_3 = \frac{7}{2}, \quad y_3 = \frac{13}{2}$$

Solving eq. (ii) & (iii)

$$x_3 = -\frac{3}{2}, \quad y_3 = \frac{3}{2}$$

So the third vertex are $\left(\frac{7}{2}, \frac{13}{2}\right)$ or $\left(-\frac{3}{2}, \frac{3}{2}\right)$

Ex.16 Find the area of quadrilateral whose vertices, taken in order, are $(-3, 2)$, $B(5, 4)$, $(7, -6)$ and $D(-5, -4)$.

Sol. Area of quadrilateral = Area of $\triangle ABC$ + Area of $\triangle ACD$

$$\text{So, Area of } \triangle ABC = \frac{1}{2} |(-3)(4+6) + 5(-6-2) + 7(2-4)|$$

$$= \frac{1}{2} |-30 - 40 - 14|$$

$$= \frac{1}{2} |-84| = 42 \text{ Sq. units}$$

$$\text{Area of } \triangle ACD = \frac{1}{2} |-3(-6+4) + 7(-4-2) + (-5)(2+6)|$$

$$= \frac{1}{2} |6 - 42 - 40| = \frac{1}{2} |-76| = 38 \text{ Sq. units}$$

$$\text{So, Area of quadrilateral ABCD} = 42 + 38 = 80 \text{ Sq. units.}$$

