

8.2 ILLUSTRATIONS

EX.1 In the given triangle $AB = 3$ cm and $AC = 5$ cm. Find all trigonometric ratios.

Sol. Using Pythagoras theorem

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow 5^2 = 3^2 + p^2$$

$$\Rightarrow 16 = p^2 \quad \Rightarrow \quad P = 4 \text{ cm}$$

Here $P = 4$ cm, $B = 3$ cm, $H = 5$ cm

$$\therefore \sin \theta = \frac{P}{H} = \frac{4}{5}$$

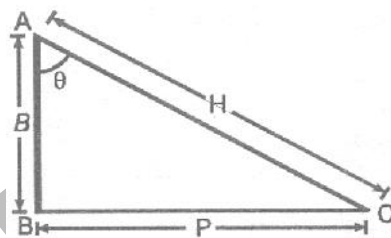
$$\cos \theta = \frac{B}{H} = \frac{3}{5}$$

$$\tan \theta = \frac{P}{B} = \frac{4}{3}$$

$$\cot \theta = \frac{B}{P} = \frac{3}{4}$$

$$\sec \theta = \frac{H}{B} = \frac{5}{3}$$

$$\operatorname{cosec} \theta = \frac{H}{P} = \frac{5}{4}$$



Ex.2 If $\tan \theta = \frac{m}{n}$, then find $\sin \theta$.

Sol. Let $P = m\alpha$ and $B = n\alpha$

$$\therefore \tan \theta = \frac{P}{B} = \frac{m}{n}$$

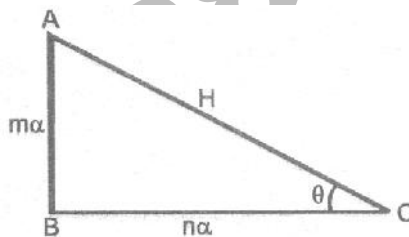
$$H^2 = P^2 + B^2$$

$$H^2 = m^2\alpha^2 + n^2\alpha^2$$

$$H = \alpha\sqrt{m^2 + n^2}$$

$$\therefore \tan \theta = \frac{P}{H} = \frac{m\alpha}{\alpha\sqrt{m^2 + n^2}}$$

$$\sin \theta = \frac{m}{\sqrt{m^2 + n^2}}$$



Ex.3 If $\operatorname{cosec} A = \frac{13}{5}$ then prove that $\tan^2 A - \sin^2 A = \sin^4 A \sec^2 A$.

Sol. We have $\cos A = \frac{13}{5} = \frac{\text{Hypotenuse}}{\text{Perpendicular}}$

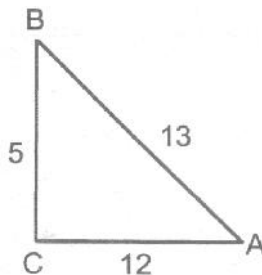
So, we draw a right triangle ABC, right angled at C such that hypotenuse AB = 13 units and perpendicular BC = 5 units

By Pythagoras theorem,

$$AB^2 = BC^2 + AC^2 \Rightarrow (13)^2 = (5)^2 + AC^2$$

$$AC^2 = 169 - 25 = 144$$

$$AC = \sqrt{144} = 12 \text{ units}$$



$$\tan A = \frac{BC}{AC} = \frac{5}{12}$$

$$\sin A = \frac{BC}{AB} = \frac{5}{13}$$

and $\sec A = \frac{AB}{AC} = \frac{13}{12}$

L.H.S. $\tan^2 A - \sin^2 A$

R.H.S. $= \sin^4 A \times \sec^2 A$

$$\begin{aligned} &= \left(\frac{5}{12}\right)^2 - \left(\frac{5}{13}\right)^2 \\ &= \frac{25}{144} - \frac{25}{169} \\ &= \frac{25(169 - 144)}{144 \times 169} \\ &= \frac{25 \times 25}{144 \times 169} \end{aligned}$$

$$\begin{aligned} &= \left(\frac{5}{13}\right)^4 \times \left(\frac{13}{12}\right)^2 \\ &= \frac{5^4 \times 13^2}{13^4 \times 12^2} \\ &= \frac{5^4}{13^2 \times 12^2} \\ &= \frac{25 \times 25}{144 \times 169} \end{aligned}$$

So, L.H.S. = R.H.S.

Hence Proved.

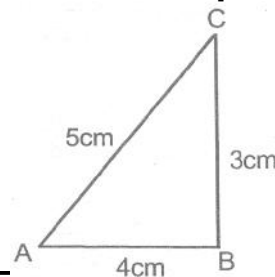
Ex.4 In $\triangle ABC$, right angled at B. $AC + AB = 9$ cm. Determine the value of $\cot C$, $\operatorname{cosec} C$, $\sec C$.

Sol. In $\triangle ABC$, we have

$$\begin{aligned} &(AC)^2 = (AB)^2 + BC^2 \\ \Rightarrow &(9 - AB)^2 = AB^2 + (3)^2 \\ \Rightarrow &(81 + AB^2 - 18AB) = AB^2 + 9 \\ \Rightarrow &72 - 18AB = 0 \\ \Rightarrow &AB = \frac{72}{18} = 4 \text{ cm.} \end{aligned}$$

$$[\because AC + AB = 9 \text{ cm} \Rightarrow AC = 9 - AB]$$

Now, $AC + AB = 9$ cm
 $AC = 9 - 4 = 5$ cm



$$\text{So, } \cot C = \frac{BC}{AB} = \frac{3}{4}, \operatorname{cosec} C = \frac{AC}{AB} = \frac{5}{4}, \sec C = \frac{AC}{BC} = \frac{5}{3}.$$

Ex.5 Given that $\cos(A - B) = \cos A \cos B + \sin A \sin B$, find the value of $\cos 15^\circ$.

Sol. Putting $A = 45^\circ$ and $B = 30^\circ$

$$\text{We get } \cos(45^\circ - 30^\circ) = \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ$$

$$\Rightarrow \cos 15^\circ = \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$$

$$\Rightarrow \cos 15^\circ = \frac{\sqrt{3} + 1}{2\sqrt{2}}$$

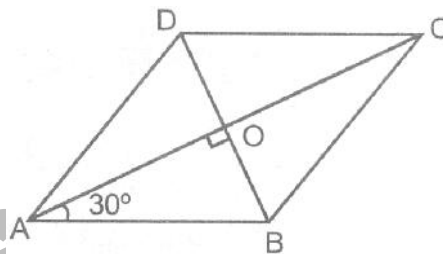
Ex.6 A Rhombus of side of 10 cm has two angles of 60° each. Find the length of diagonals and also find its area.

Sol. Let ABCD be a rhombus of side 10 cm and $\angle BAD = \angle BCD = 60^\circ$. Diagonals of parallelogram bisect each other.

So, $AO = OC$ and $BO = OD$

In right triangle AOB

$$\begin{aligned} \sin 30^\circ &= \frac{OB}{AB} & \cos 30^\circ &= \frac{OA}{AB} \\ \Rightarrow \frac{1}{2} &= \frac{OB}{10} & \Rightarrow \frac{\sqrt{3}}{2} &= \frac{OA}{10} \\ \Rightarrow OB &= 5 \text{ cm} & \Rightarrow OA &= 5\sqrt{3} \\ \therefore BD &= 2(OB) & \Rightarrow AC &= 2(OA) \\ \Rightarrow BD &= 2(5) & \Rightarrow AC &= 2(5\sqrt{3}) \\ \Rightarrow BD &= 10 \text{ cm} & \Rightarrow AC &= 10\sqrt{3} \text{ cm} \end{aligned}$$



So, the length of diagonals $AC = 10\sqrt{3}$ cm & $BD = 10$ cm

$$\begin{aligned} \text{Area of Rhombus} &= \frac{1}{2} \times AC \times BD \\ &= \frac{1}{2} \times 10\sqrt{3} \times 10 \\ &= 50\sqrt{3} \text{ cm}^2. \end{aligned}$$

Ex.7 Evaluate : $\frac{\sec^2 54^\circ - \cot^2 36^\circ}{\cos \sec^2 57^\circ - \tan^2 33^\circ} + 2 \sin^2 38^\circ \sec^2 52^\circ - \sin^2 45^\circ + \frac{2}{\sqrt{3}} \tan 17^\circ \tan 60^\circ \tan 73^\circ$

$$\text{Sol. } \frac{\sec^2 54^\circ - \cot^2 36^\circ}{\cos \sec^2 57^\circ - \tan^2 33^\circ} + 2 \sin^2 38^\circ \sec^2 52^\circ - \sin^2 45^\circ + \frac{2}{\sqrt{3}} \tan 17^\circ \tan 60^\circ \tan 73^\circ$$

$$\begin{aligned}
&= \frac{\sec^2(90^\circ - 36^\circ) - \cot^2 36^\circ}{\cos \sec^2(90^\circ - 33^\circ) - \tan^2 33^\circ} + 2 \sin^2 38^\circ \sec^2(90^\circ - 38^\circ) - \sin^2 45^\circ + \frac{2}{\sqrt{3}} \tan(90^\circ - 73^\circ) \tan 73^\circ \tan 60^\circ \\
&= \frac{\cos \sec^2 36^\circ - \cot^2 36^\circ}{\sec^2 33^\circ - \tan^2 33^\circ} + 2 \sin^2 38^\circ \cos \sec^2 38^\circ - \left(\frac{1}{\sqrt{2}}\right)^2 + \frac{2}{\sqrt{3}} \cot 73^\circ \tan 73^\circ \times \sqrt{3} \\
&= \frac{1}{1} + 2 \sin^2 38^\circ \times \frac{1}{\sin^2 38^\circ} - \frac{1}{2} + \frac{2}{\sqrt{3}} \times \frac{1}{\tan 73^\circ} \times 73^\circ \times \sqrt{3} [\because \cos \sec^2 \theta - \cot^2 \theta = 1, \sec^2 \theta - \tan^2 \theta = 1] \\
&= 1 + 2 - \frac{1}{2} + 2 = 5 - \frac{1}{2} \\
&= \frac{9}{2}.
\end{aligned}$$

Ex.8 Prove that : $\cos \sec(65^\circ + \theta) - \sec(25^\circ - \theta) - \tan(55^\circ - \theta) + \cot(35^\circ + \theta) = 0$

Sol. $\cos \sec(65^\circ + \theta) = \cos \sec\{90^\circ - (25^\circ - \theta)\} = \sec(25^\circ - \theta)$ (i)
 $\cot(35^\circ + \theta) = \cot\{90^\circ - (55^\circ - \theta)\} = \tan(55^\circ - \theta)$ (ii)
 $\therefore$ **L.H.S.** $\cos \sec(65^\circ + \theta) - \sec(25^\circ - \theta) - \tan(55^\circ - \theta) + \cot(35^\circ + \theta)$
 $= \sec(25^\circ - \theta) - \sec(25^\circ - \theta) - \tan(55^\circ - \theta) + \tan(55^\circ - \theta)$
 $= 0$ [using (i) & (ii)] **R.H.S.**

Ex.9 Prove that : $\cot \theta - \tan \theta = \frac{2 \cos^2 \theta - 1}{\sin \theta \cos \theta}$

Sol. **L.H.S.** $\cot \theta - \tan \theta$

$$\begin{aligned}
&= \frac{\cos \theta}{\sin \theta} - \frac{\sin \theta}{\cos \theta} \quad \left[\because \cot \theta = \frac{\cos \theta}{\sin \theta}, \tan \theta = \frac{\sin \theta}{\cos \theta} \right] \\
&= \frac{\cos^2 \theta - \sin^2 \theta}{\sin \theta \cos \theta} = \frac{\cos^2 \theta - (1 - \cos^2 \theta)}{\sin \theta \cos \theta} \quad [\because \sin^2 \theta = 1 - \cos^2 \theta] \\
&= \frac{\cos^2 \theta - 1 + \cos^2 \theta}{\sin \theta \cos \theta} = \frac{2 \cos^2 \theta - 1}{\sin \theta \cos \theta}
\end{aligned}$$

R.H.S.

Hence Proved.

Ex.10 Prove that : $(\csc A - \sin A)(\sec A - \cos A)(\tan A + \cot A) = 1$.

Sol. **L.H.S.** $(\csc A - \sin A)(\sec A - \cos A)(\tan A + \cot A)$

$$\begin{aligned}
&= \left(\frac{1}{\sin A} - \sin A \right) \left(\frac{1}{\cos A} - \cos A \right) \left(\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A} \right) \\
&= \left(\frac{1 - \sin^2 A}{\sin A} \right) \left(\frac{1 - \cos^2 A}{\cos A} \right) \left(\frac{\sin^2 A + \cos^2 A}{\sin A \cos A} \right) \\
&= \left(\frac{\cos^2 A}{\sin A} \right) \left(\frac{\sin^2 A}{\cos A} \right) \left(\frac{1}{\sin A \cos A} \right) \quad [\because \sin^2 A + \cos^2 A = 1]
\end{aligned}$$

= 1

R.H.S.

Hence Proved.

Ex. 11 If $\sin \theta + \cos \theta = m$ and $\sec \theta + \csc \theta = n$, then prove that $n(m^2 - 1) = 2m$.

Sol. **L.H.S.** $n(m^2 - 1)$

$$= (\sec \theta + \csc \theta)[(\sin \theta + \cos \theta)^2 - 1]$$

$$= \left(\frac{1}{\cos \theta} + \frac{1}{\sin \theta} \right) (\sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta - 1)$$

$$= \left(\frac{\cos \theta + \sin \theta}{\sin \theta \cos \theta} \right) (1 + 2 \sin \theta \cos \theta - 1)$$

$$= \frac{(\cos \theta + \sin \theta)}{\sin \theta \cos \theta} (2 \sin \theta \cos \theta)$$

$$= 2(\sin \theta + \cos \theta)$$

$$= 2m$$

R.H.S.

Hence Proved.

Ex.12 If $\sec \theta = x + \frac{1}{4x}$, then prove that $\sec \theta + \tan \theta = 2x$ or $\frac{1}{2x}$.

Sol. $\sec \theta = x + \frac{1}{4x}$ (i)

$$\therefore 1 + \tan^2 \theta = \sec^2 \theta$$

$$\Rightarrow \tan^2 \theta = \sec^2 \theta - 1 \Rightarrow \tan^2 \theta = \left(x + \frac{1}{4x} \right)^2 - 1$$

$$\Rightarrow \tan^2 \theta = x^2 + \frac{1}{16x^2} + 2 \times x \times \frac{1}{4x} - 1$$

$$\Rightarrow \tan^2 \theta = x^2 + \frac{1}{16x^2} + \frac{1}{2} - 1 \Rightarrow \tan \theta = \pm \left(x - \frac{1}{4x} \right)$$

$$\Rightarrow \tan^2 \theta = \left(x - \frac{1}{4x} \right)^2 \Rightarrow \tan \theta = \pm \left(x - \frac{1}{4x} \right)$$

$$\text{So, } \tan \theta = x - \frac{1}{4x} \text{(ii)}$$

$$\text{or } \tan \theta = -\left(x - \frac{1}{4x} \right) \text{(iii)}$$

Adding equation (i) and (ii)

$$\sec \theta + \tan \theta = x + \frac{1}{4x} + x - \frac{1}{4x}$$

$$\sec \theta + \tan \theta = 2x$$

Adding equation (i) and (ii)

$$\sec \theta + \tan \theta = x + \frac{1}{4x} - x + \frac{1}{4x}$$

$$= \frac{1}{2x}$$

Hence, $\sec \theta + \tan \theta = 2x$ or $\frac{1}{2x}$.

Ex.13 If θ is an acute angle and $\tan \theta + \cot \theta = 2$ find the value of $\tan^9 \theta + \cot^9 \theta$

Sol. We have, $\tan \theta + \cot \theta = 2$

$$\Rightarrow \tan \theta + \frac{1}{\tan \theta} = 2$$

$$\Rightarrow \frac{\tan^2 \theta + 1}{\tan \theta} = 2$$

$$\Rightarrow \tan^2 \theta + 1 = 2 \tan \theta$$

$$\Rightarrow \tan^2 \theta - 2 \tan \theta + 1 = 0$$

$$\Rightarrow (\tan \theta - 1)^2 = 0$$

$$\Rightarrow \tan \theta - 1 = 0$$

$$\Rightarrow \tan \theta = 1$$

$$\Rightarrow \tan \theta = \tan 45^\circ$$

$$\Rightarrow \theta = 45^\circ$$

$$\therefore \tan^9 \theta + \cot^9 \theta$$

$$= \tan^9 45^\circ + \cot^9 45^\circ$$

$$= (\tan 45^\circ)^9 + (\cot 45^\circ)^9$$

$$= (1)^9 + (1)^9$$

$$= 2.$$