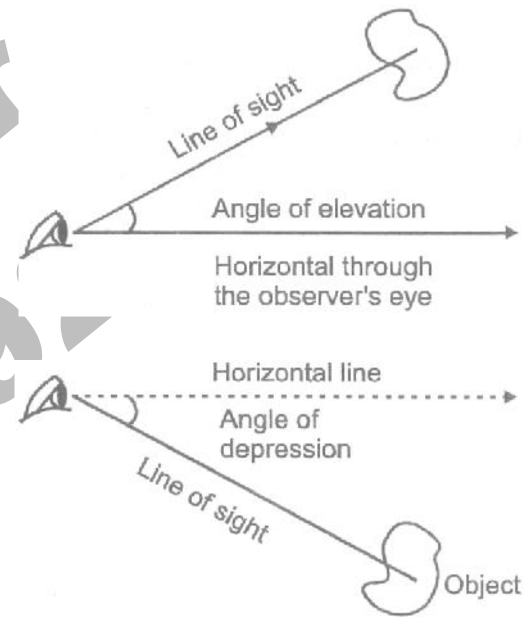
CHAPTER – 9
HEIGHTS & DISTANCES

9.1 INTRODUCTION

ANGLE OF ELEVATION:

In order to see an object which is at a higher level compared to the ground level we are to look up. The line joining the object and the eye of the observer is known as the line sight and the angle which this line of sight makes with the horizontal drawn through the eye of the observer is known as the angle of elevation. Therefore, the angle of elevation of an object helps in finding out its height (**figure)**



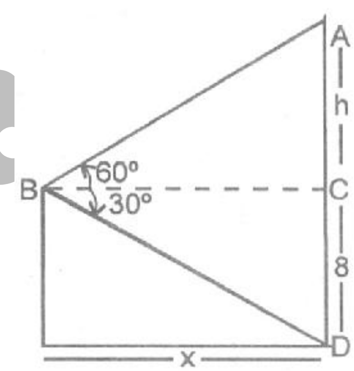
ANGLE OF DEPRESSION:

When the object is at a lower level tan the observer's eyes, he has to look downwards to have a view of the object. It that case, the angle which the line of sight makes with the horizontal thought the observer's eye is known as the **angle of depression (Figure).**

ILLUSTRATIONS:

EX.1 A man is standing on the deck of a ship, which is 8 m above water level. He observes the angle of elevations of the top of a hill as 60^0 and the angle of depression of the base of the hill as 30^0. Calculation the distance of the hill from the ship and the height of the hill.

Sol. Let x be distance of hill from man and h + 8 be height of hill which is required.
is right triangle ACB.



$\Rightarrow \quad \tan 60^0 = \frac{AC}{BC} = \frac{h}{x}$

$\Rightarrow \quad \sqrt{3} = \frac{h}{x}$

In right triangle BCD.

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{8}{x} \quad \Rightarrow \quad x = 8\sqrt{3}$

$\therefore$ Height of hill = h + 8 = $\sqrt{3}.x + 8 = (\sqrt{3})(8\sqrt{3}) + 8 = 32$ m.

Distance of ship from hill = x = $8\sqrt{3}$ m.

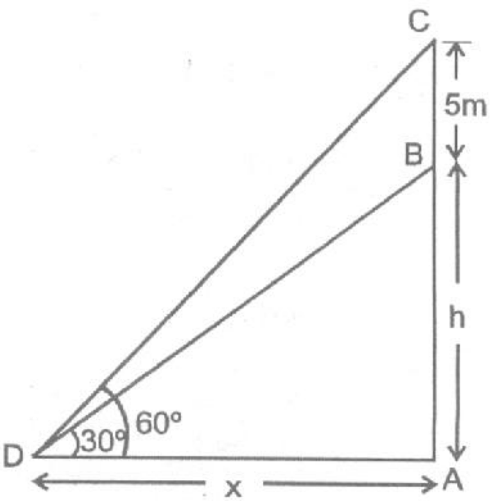
Ex.2 A vertical tower stands on a horizontal plane and is surmounted by vertical flag staff of height 5 meters. At a point on the plane, the angle of elevation of the bottom and the top of the flag staff are respectively 30^0 and 60^0 find the height of tower. **[CBSE-2006]**

Sol. Let AB be the tower of height h metre and BC be the height of flag staff surmounted on the tower, Let the point of the place be D at a distance x meter from the foot of the tower in Δ ABD

$$\tan 30^0 = \frac{AB}{AD}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{x}$$

$$\Rightarrow \quad x = \sqrt{3}h \qquad \text{.....(i)}$$



In Δ ABD

$$\tan 60^0 = \frac{AC}{AD}$$

$$\Rightarrow \quad \sqrt{3} = \frac{5+h}{x}$$

$$\Rightarrow \quad x = \frac{5+h}{\sqrt{3}} \qquad \text{.......(ii)}$$

From (i) and (ii)

$$\Rightarrow \quad \sqrt{3}\ h\frac{5+h}{\sqrt{3}}$$

$$\Rightarrow \quad 3h = 5+h$$

$$\Rightarrow \quad 2h = 5$$

$$\Rightarrow \quad h = \frac{5}{2} = 2.5m$$

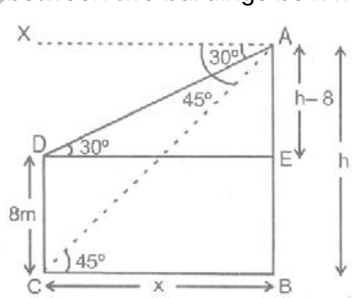
So, the height of tower = 2.5 m

Ex.3 The angles of depressions of the top and bottom of 8m tall building from the top of a multistoried building are 30^0 and 45^0 respectively. Find the height of multistoried building and the distance between the two buildings.

Sol. Let AB be the multistoried building of height h and let the distance between two buildings be x meters.

$\angle XAC = \angle ACB = 45^0$ [Alternate angles $\because$ AX $\parallel$ DE]

$XAD = ADE = 30^0$ [Alternate angles $\because$ AX $\parallel$ BC]



In Δ ADE

$$\tan 30^0 = \frac{AE}{ED}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h-8}{x} \qquad (\because CB = DE = x)$$

$$\Rightarrow \quad x = \sqrt{3}(h-8) \qquad \text{.......(i)}$$

In Δ ACB

$\tan 45^0 = \frac{h}{x}$

$\Rightarrow \quad 1 = \frac{h}{x}$

$\Rightarrow \quad x = h$ ....(ii)

Form (i) and (ii)

$\sqrt{3}(h-8) = h \quad \Rightarrow \quad \sqrt{3}h - 8\sqrt{3} = h$

$\Rightarrow \quad \sqrt{3}h - h = 8\sqrt{3}$

$\Rightarrow \quad h(\sqrt{3}-1) = 8\sqrt{3}$

$\Rightarrow \quad h = \frac{8\sqrt{3}}{\sqrt{3}-1} \times \frac{(\sqrt{3}+1)}{\sqrt{3}+1}$

$\Rightarrow \quad h = \frac{8\sqrt{3}(\sqrt{3}+1)}{2}$

$\Rightarrow \quad h = 4\sqrt{3}(\sqrt{3}+1)$

$\Rightarrow \quad h = 4(3+\sqrt{3})$ metres

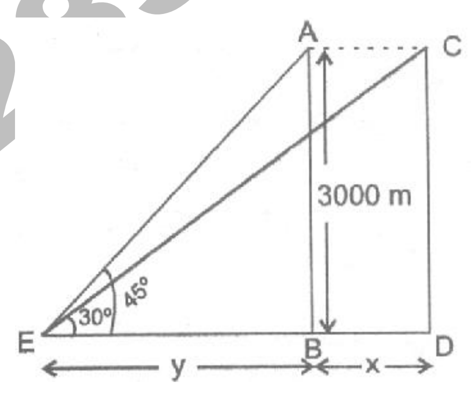
Form (ii) x = h

So, x = 4(3 + $\sqrt{3}$) metres

Hence, height of multistoried building = 4(3 + $\sqrt{3}$) metres
Distance between two building = 4(3 + $\sqrt{3}$) metres

Ex.4 The angle of elevation of an aeroplane from a point on the ground is 45^0. After a flight of 15 sec, the elevation changes to 30^0. If the aeroplane is flying at a height of 3000 metres, find the speed of the aeroplane.

Sol. Let the point on the ground is E which is y metres from point B and let after 15 sec flight it covers x metres distance.
In Δ AEB.



$\tan 45^0 = \frac{AB}{EB}$

$\Rightarrow \quad 1 = \frac{3000}{y}$

$\Rightarrow \quad y = 3000\,m$ ......(i)

In Δ CED

$\Rightarrow \quad \tan 30^0 = \frac{CD}{ED}$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{3000}{x+y} \qquad (\because AB = CD)$

$\Rightarrow \quad x + y = 3000\sqrt{3} \qquad \ldots\ldots(ii)$

From equation (i) and (ii)

$\Rightarrow \quad x + 3000 = 3000\sqrt{3}$

$\Rightarrow \quad x = 3000\sqrt{3} - 3000$

$\Rightarrow \quad x = 3000(\sqrt{3} - 1)$

$\Rightarrow \quad x = 3000 \times (1.732 - 1)$

$\Rightarrow \quad x = 2196\,m$

Speed of Aeroplane $= \frac{\text{Distance covered}}{\text{Tiem taken}}$

$= \frac{2196}{15} m/sec. = 146.4\ m/sec.$

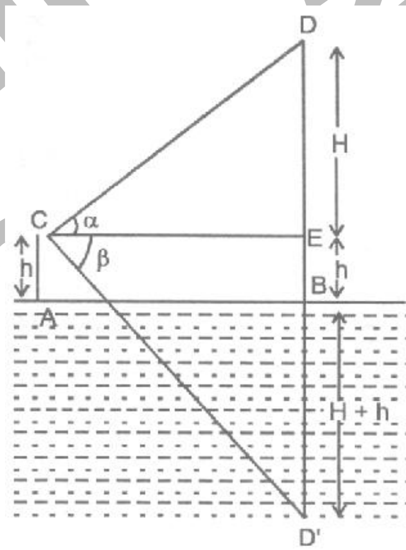
$= \frac{2196}{15} \times \frac{18}{5} Km/hr$

= 527.04 Km/hr

Hence, the speed of aeroplane is 527.04 Km/hr.

Ex.5 If the angle of elevation of cloud from a point h metres above a lake is α and the angle of depression of its reflection in the lake is β, prove that the distance of the cloud from the point of observation is $\frac{2h \sec \alpha}{\tan\beta - \tan\alpha}$.

Sol. Let AB be the surface of the lake and let C be a point of observation such that AC- h metres. Let D be the position of the cloud and D' be its reflection in the lake. Then BD = BD'.
In ΔDCE



$\tan\alpha = \frac{DE}{CE}$

$\Rightarrow \quad CE = \frac{H}{\tan\alpha} \qquad \ldots\ldots(i)$

In $\Delta CED'$

$\tan\beta = \frac{ED'}{EC}$

$\Rightarrow \quad CE = \frac{h + H + h}{\tan\beta}$

$\Rightarrow \quad CE = \frac{2h + H}{\tan\beta} \qquad \ldots\ldots(ii)$

From (i) & (ii)

$\Rightarrow \quad \frac{H}{\tan\alpha} = \frac{2h+H}{\tan\beta}$

$\Rightarrow \quad H\tan\beta = 2h\ \tan\alpha + H\tan\alpha$

$\Rightarrow \quad H\tan\beta - H\ \tan\alpha + 2h\tan\alpha$

$\Rightarrow \quad H(\tan\beta - \tan\alpha) = 2h\ \tan\alpha$

$\Rightarrow \quad H = \frac{2h\ \tan\alpha}{\tan\beta - \tan\alpha}$ ....(iii)

In ΔDCE

$\text{Sin}\ \alpha = \frac{DE}{CD}$

$\Rightarrow \quad CD = \frac{DE}{\sin\alpha} \quad \Rightarrow \quad CD = \frac{H}{\sin\alpha}$

Substituting the value of H from (iii)

$CD = \frac{2h\ \tan\alpha}{(\tan\beta - \tan\alpha)\sin\alpha} \quad \Rightarrow \quad CD = \frac{2h\frac{\sin\alpha}{\cos\alpha}}{(\tan\beta - \tan\alpha)\sin\alpha}$

$CD = \frac{2h\ \tan\alpha}{\tan\beta - \tan\alpha}$

Hence, the distance of the cloud from the point of observation is $\frac{2h\ \sec\alpha}{\tan\beta - \tan\alpha}$ **Hence Proved.**

Ex.6 A boy is standing on the ground and flying a kite with 100 m of string at an elevation of 30^0. Another boy is standing on the roof of a 10 m high building and is flying his kite at an elevation of 45^0. Both the boys are on opposite sides of both the kites. Find the length of the string that the second boy must have so that the two kites meet.

Sol. Let the length of second string b x m.

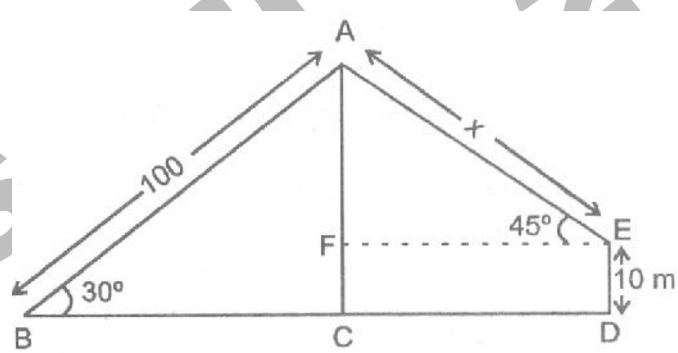
In ΔABC

$\text{Sin}\ 30^0 = \frac{AC}{AB}$

$\frac{1}{2} = \frac{AC}{100} \Rightarrow AC = 50\,m$

In ΔAEF

$\text{Sin}\ 30^0 = \frac{AF}{AE}$



$\frac{1}{\sqrt{2}} = \frac{AC - FC}{x}$

$\frac{1}{\sqrt{2}} = \frac{50-10}{x}$ [$\because$ AC = 50 m, FC = ED = 10 m]

$\frac{1}{\sqrt{2}} = \frac{40}{x}$

$x = 40\sqrt{2}m$

(So the length of string that the second boy must have so that the two kites meet = $40\sqrt{2}$ m.)

Chapter 9

ASSIGNMENT

OBJECTIVE EXCERCISE - 9.1

1. Upper part of a vertical tree which is broken over by the winds just touches the ground and makes an angle of 30^0 with the ground. If the length of the broken part is 20 metres, then the remaining part of the trees is of length

(A) 20 metres (B) $10\sqrt{3}$ metres (C) 10 metres (D) $10\sqrt{2}$ metres

2. The angle of elevation of the top of a tower as observed from a point on the horizontal ground is 'x'. If we move a distance 'd' towards the foot of the tower, the angle of elevation increases to 'y', then the height of the tower is

(A) $\frac{d \tan x \tan y}{\tan y - \tan x}$ (B) d(tan y + tan x) (C) d(tan y – tan x) (D) $\frac{d \tan x \tan y}{\tan y + \tan x}$

3. The angle of elevation of the top of a tower, as seen from two points A & B situated in he same line and at distances 'p' and 'q' respectively from the foot of the tower, are complementary, then the height of the tower is

(A) pq (B) $\frac{p}{q}$ (C) $\sqrt{pq}$ (D) noen of these

4. The angle of elevation of the top of a tower at a distance of $\frac{50\sqrt{3}}{3}$ metres from the foot is 60^0. Find the height of the tower

(A) $50\sqrt{3}$ metres (B) $\frac{20}{\sqrt{3}}$ metres (C) -50 metres (D) 50 metres

5. The Shadow of a tower, when the angle of elevation of the sun is 30^0, is found to be 5 m longer than when its was 45^0, then the height of tower in metre is

(A) $\frac{5}{\sqrt{3}+1}$ (B) $\frac{5}{2}(\sqrt{3}-1)$ (C) $\frac{5}{2}(\sqrt{3}+1)$ (D) None of these.

SUBJECTIVE EXCERCISE - 9.2

1. From the top a light house, the angles of depression of two ships of the opposite sides of it are observed to be α and β. If the height of the light house be h meters and the line joining the ships passes thought the foot of the light house. Show that the distance between the ships is $\frac{h(\tan\alpha + \tan\beta)}{\tan\alpha \tan\beta}$ meters.

2. A ladder rests against a wall at angle α to the horizontal. Its foot is pulled away from the previous point through a distance 'a', so that is slides down a distance 'b' on the wall making an angle β. With the horizontal show that $\frac{a}{b} = \frac{\cos\alpha - \cos\beta}{\sin\beta - \sin\alpha}$