ANSWER PAPER- 1

1. In how many of the distinct permutations of the letters in MISSISSIPPI do the four I's not come together?

Correct Answer :

Explanation:

Total letters of the word MISSISSIPPI = 11

Here M=1, I = 4, S = 4 and P = 2

$\therefore$ Number of permutations = $\frac{11!}{4!4!2!}$

$= \frac{11\times 10\times 9\times 8\times 7\times 6\times 5\times 4!}{4!\times 4\times 3\times 2\times 1\times 2\times 1}$ = 34650

when the four I's come together then it becomes one letter so total number of letters in the word when all I's come together = 8.

$\therefore$ Number of Permutations = $\frac{8!}{4!2!} = \frac{8\times 7\times 6\times 5\times 4!}{4!\times 2\times 1}$ = 840

Number of permutations when four I's do not come together $= 34650 - 840$ = 33810.

2. Find the modulus and the arguments of the complex number $z = -\sqrt{3} + i$

Correct Answer :

Explanation:

Here $z = -\sqrt{3} + i$ = $r(\cos\theta + i\sin\theta)$

$\Rightarrow r\cos\theta = -\sqrt{3}$ and $r\sin\theta = 1$

Squaring both sides of (i) and adding

$r^2(\cos^2\theta + \sin^2\theta)$ = 3 + 1 $\Rightarrow r^2 = 4$ $\Rightarrow$r = 2

$\therefore$ $2\cos\theta = -\sqrt{3}$ and $2\sin\theta = 1$

$\Rightarrow \cos\theta = \frac{-\sqrt{3}}{2}$ and $\sin\theta = \frac{1}{2}$

Since $\sin\theta$ is positive and $\cos\theta$ is negative

$\therefore$ θ lies in second quadrant

$\therefore$ $\theta = \left(\pi - \frac{\pi}{6}\right) = \frac{5\pi}{6}$

$\therefore$ |z| = 2 and arg (z) $= \frac{5\pi}{6}$

3. For any two complex numbers z_1 and z_2 prove that

$Re(z_1 z_2) = Re\, z_1\, Re\, z_2 - Im\, z_1\, Im\, z_2.$

Correct Answer :

Explanation:

Let $z_1 = a_1 + ib_1$ and $z_2 = a_2 + ib_2$

Then $\mathrm{Re}(z_1) = a_1, \mathrm{Re}(z_2) = a_2, \mathrm{Im}(z_1) = b_1$ and $\mathrm{Im}(z_2) = b_2$

Now $z_1 z_2 = (a_1 + ib_1)(a_2 + ib_2)$

$= a_1a_2 + ia_1b_2 + ia_2b_1 + i^2 b_1b_2$

$= (a_1a_2 - b_1b_2) + i(a_1b_2 + a_2b_1)$ $(\because i^2 = -1)$

$\mathrm{Re}(z_1z_2) = a_1a_2 - b_1b_2$

$= \mathrm{Re}(z_1)\,\mathrm{Re}(z_2) - \mathrm{Im}(z_1)\,\mathrm{Im}(z_2)$

4. Prove $\dfrac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} = \cot 3x$

Correct Answer :

Explanation:

We have L.H.S. = $\dfrac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} = \dfrac{(\cos 4x + \cos 2x) + \cos 3x}{(\sin 4x + \sin 2x) + \sin 3x}$

$$= \frac{2\cos\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right) + \cos 3x}{2\sin\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right) + \sin 3x}$$

$$\left[\begin{array}{l}\because \sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)\\ \cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)\end{array}\right]$$

$$= \frac{2\cos 3x\cos x + \cos 3x}{2\sin 3x\cos x + \sin 3x}$$

$= \dfrac{\cos 3x\,(2\cos x + 1)}{\sin 3x\,(2\cos x + 1)} = \dfrac{\cos 3x}{\sin 3x}$ = cot 3x = R.H.S

5. Let A = {x, y, z} and B = {1, 2}. Find the number of relations from A to B.

Correct Answer :

Explanation:

Here A ={x, y, z} and B = {1, 2}

Number of elements in set A = 3

Number of elements in set B = 2

Number of subsets of $A \times B = 3 \times 2 = 6$

Number of relations from A to B $= 2^6$

Hint:

6. Let A = {9, 10, 11, 12, 13}and let $\because \quad 9 = 3 \times 3$ be defined by f (n) = the highest prime factor of n. Find the range of f.

Correct Answer :

Explanation:

Here A = {9, 10, 11, 12, 13}

For n = 9, f (9) = 3($\because \quad 9 = 3 \times 3$ and 3 is highest prime factor of 9}

For n = 10, f (10) = 5 $(\because \quad 10 = 2 \times 5)$

For n = 11, f (11) = 11 $(\because \quad 11 = 1 \times 11)$

For n = 12, f (12) = 3 $(\because \quad 12 = 3 \times 2 \times 2)$

For n = 13, f (13) = 13 $(\because \quad 13 = 1 \times 13)$

$\therefore$ Range of f - {5, 11, 3, 13}

= {3, 5, 11, 13}

7. In how many ways can the letters of the word ASSASSINATION be arranged so that all the S's are together?

Correct Answer :

Explanation:

Here total letters are 13 in the word ASSASSINATION in which A appears 3 times, S appears 4 times, 1 appears 2 times and N appears 2 times. Now four S's taken together become a single letter and other remaining letters taken with this single letter.

$\therefore$ Number of arrangements $= \frac{10!}{3!2!2!} = \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3!}{3! 2 \times 1 \times 2 \times 1}$

$= 10 \times 9 \times 8 \times 7 \times 6 \times 5$ = 151200

8. Solve the following system of inequalities graphically:
$2x + y \geq 4$, $x + y \leq 3$, $2x - 3y \leq 6$

Correct Answer :

Explanation:

The given inequality is $2x + y \geq 4$

Draw the graph of the line 2x + y = 4

Table of values satisfying the equation 2x + y = 4

x	2	1
y	0	2

Putting (0, 0) in the given inequation, we have

$2 \times 0 + 0 \geq 0 \Rightarrow \quad 0 \geq 4$, which is false.

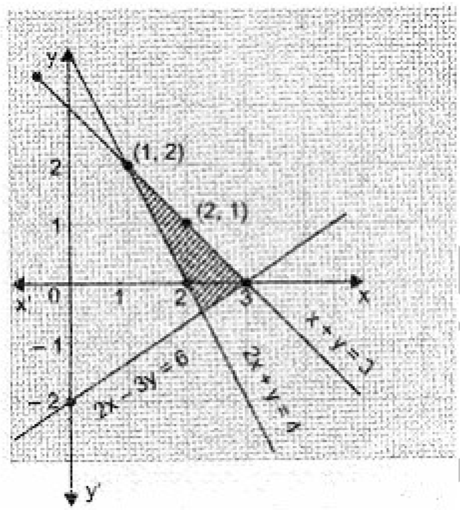
$\therefore$ Half plane of $2x + y \geq 4$ is away from origin.

Also the given inequality is $x + y \leq 3$

Draw the graph of the line x + y = 3

Table of values satisfying the equation x + y = 3

x	2	1
y	1	2



Putting (0, 0) in the given inequation, we have
$0+0 \le 3 \Rightarrow 0 \le 3$, which is true
$\therefore$ Half plane of $x+y \le 3$ is towards origin.
The given inequality is $2x-3y \le 6$
Draw the graph of the line $2x-3y=6$
Table of values satisfying the equation $2x-3y=6$

x	0	3
y	−2	0

Putting (0, 0) in the given inequation, we have
$2\times 0-3\times 0 \le 6 \Rightarrow 0 \le 6$, which is true,
$\therefore$ Half plane of $2x-3y \le 6$ is towards origin.

Hint:

9. Prove that : cos A cos(60 - A) cos (60 + A) = $\frac{1}{4}\cos 3A$

Correct Answer :

Hint:

10. Prove that : tan a + 2 tan 2a + 4 tan 4a + 8 cot 8a = cot a

Correct Answer :

Hint:

11. A college awarded 38 medals in Football, 15 in Basketball and 20 to Cricket. If, these medals went to a total of 58 men and only three men got medals in all the three sports, how many received medals in exactly two of the three sports ?

Correct Answer :

Hint: